University of Global Village (UGV), Barishal

Electrical Machine II

Content of Theory Course

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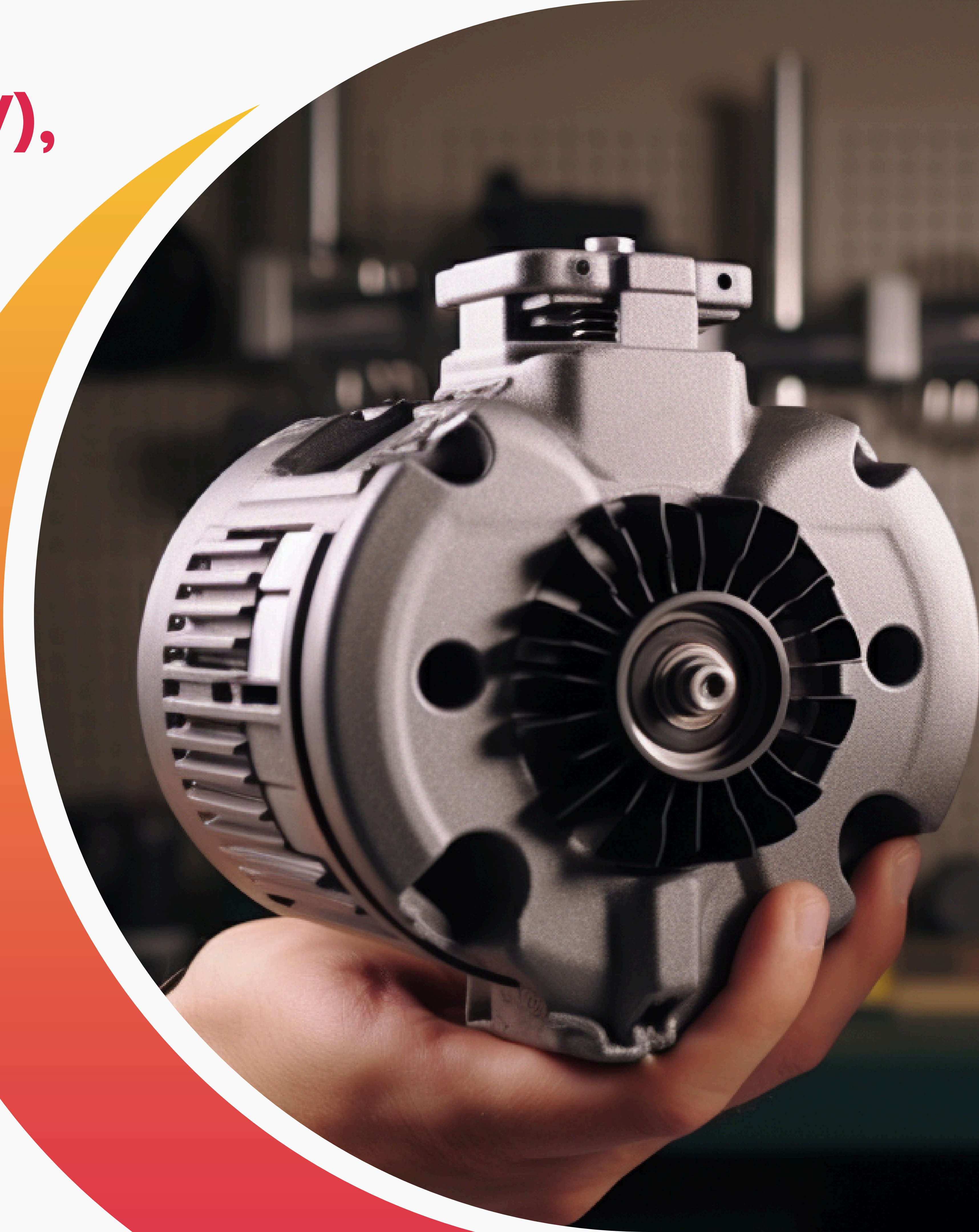


Basic Course Information

Course Title	Electrical Machine - II
Course Code	EEE- 0714-2203
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	4th Semester
Academic Session	Winter 2025

Continuous Assessment Pattern

Quizzes

Altogether 4 quizzes may be taken during the semester, 2 quizzes will be taken for midterm and 2 quizzes will be taken for final term.

Assignment

Altogether 4 assignments may be taken during the semester, 2 assignments will be taken for midterm and 2 assignments will be taken for final term.

Presentation

The students will have to form a group of maximum 3 members. The topic of the presentation will be given to each group and students will have to do the group presentation on the given topic.

Assessment Pattern

CIE – Continuous Internal Evaluation (90 Marks)

Bloom's Category Marks (out of 90)	Mid Exam (45)	Assignment (15)	Quiz (15)	Attendance & External Participation in Curricular/Co-Curricular Activities (15)
Remember	05		05	
Understand	05	05	05	
Apply	10		05	15
Analyze	10			
Evaluate	10			
Create	05	05		

Assessment Pattern

SEE – Semester End Examination (60 Marks)

Bloom's Category	Final Examination
Remember	15
Understand	10
Apply	10
Analyze	10
Evaluate	10
Create	05

Course Learning Outcomes (CLOs)

After completing this course successfully, the students will be able to -

CLO 1

Explain the aspects of construction, principles of operations and applications of electrical machines.

CLO 2

Execute performance analysis of electrical machines.

CLO 3

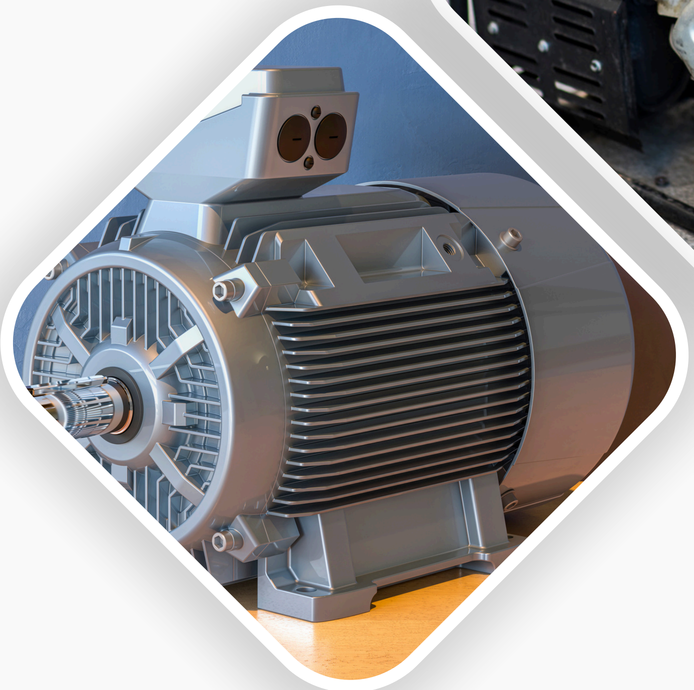
Design electrical machines subject to specific requirements.

CLO 4

Conduct experiments for analysis of single and three phase electric machine performance.

Synopsis / Rationale

This Course is essential for the students to learn basic principles and the operation of AC electrical machines such as synchronous generator, synchronous motor, three phase and single-phase induction motors. By understanding these machines' theoretical foundations and practical applications, students gain insights into efficient power generation, transmission, and utilization. Furthermore, the course equips learners with the skills to analyze, design, and troubleshoot synchronous and induction machines, fostering expertise crucial for electrical engineering and energy-related industries



Course Objectives

- To learn about the basic electrical machines, their operation, and applications.
- To understand the basic principle and operation of electrical machines like synchronous generator, synchronous motor, and induction motor.
- To become acquainted with the applications of these machines in the electrical power system.

Course Summary

Serial No.	Course Content	Hours
1.	Synchronous machines- types-constructional details-types of armature windings- double layer -integral & fractional slot – lap – single layer – hemitropic, whole-coil & mush windings - (developed winding diagram for assignment only.) - Principle of operation - synchronous generators - EMF equation-winding factor- space and time harmonics - flux density distribution and their analysis- armature reaction in 3-phase and 1-phase alternators - leakage reactance - synchronous reactance - pharos diagram under loaded condition - load characteristics.	**15**
2.	Voltage regulation of alternators- methods for finding regulation - emf, mmf, Potier methods-salient pole machines - two reaction theory – regulation by slip test- reluctance power - short circuit conditions-concept of transient and sub transient reactance’s - parallel operation of alternators- synchronizing methods - two alternators in parallel – governor characteristics – load sharing - synchronizing power- operation on infinite bus bars - torque angle - maximum power – power angle diagram – methods of excitation - automatic voltage regulators.	**15**

Course Summary

Serial No.	Course Content	Hours
3.	Synchronous motors- principle of operation- operation on infinite bus bars - phasor diagrams constant excitation and constant power output circle diagram - V curves and inverted V curves for motor and generator operations - hunting and suppression - starting methods – synchronous condenser – applications of synchronous motor.	**15**
4.	Design of synchronous machines - specific loading- choice of specific electric and magnetic loadings – output equation-classification-turbo alternators- water wheel generators- -separation of D and L- main dimensions - short circuit ratio and its importance in design.	**15**

Course Plan Mapped with CLO

Week No.	Topics	Teaching-Learning Strategy	Assessment Strategy	Alignment to CLO
1.	Basic Principle of Alternator Operation, Details about Stationary Armature, Details of Construction, Damper Windings, Speed and Frequency	Lecture, Multimedia, Group Discussion	Feedback, Q&A	CLO 1
2.	Armature Windings, Concentric or Chain Windings, Two layer Winding, Wye and Delta Connections, Short Pitch Winding, Pitch factor/Chording factor—Distribution or Breadth Factor or Winding Factor or Spread Factor	Lecture, Multimedia, Practical Example	Feedback, Q&A	CLO 1 CLO 2
3.	Equation of Induced E.M.F, Effect of Hormonics on Pitch and Distribution Factors, Factors Affecting Alternator Size,Vector Determination of Voltage Regulation	Lecture, Multimedia, Practical Example	Feedback, Q&A	CLO 2 CLO 3

Course Plan Mapped with CLO

Week No.	Topics	Teaching-Learning Strategy	Assessment Strategy	Alignment to CLO
4.	Operation of Salient Pole Synchronous Machine, Phasor Diagram for a Salient Pole Synchronous Machine, Calculations from Phasor Diagram	Lecture, Multimedia, Group Discussion	Midterm Quiz #1	CLO 3
5.	Power Developed by a Synchonous Generator, Conditions for Parallel operation of Alternators	Lecture, Multimedia,	Feedback, Q&A , Assignment	CLO 4
6.	Synchronizing of Alternators, Synchronizing Current, Synchronizing Power,Alternators Connected to Infinite Busbars, Synchronizing Torque, Effect of Load on Synchronizing Power, Alternative Expression for Synchronizing Power	Lecture, Multimedia, Practical Example	Feedback, Q&A	CLO 3 CLO 4

Course Plan Mapped with CLO

Week No.	Topics	Teaching-Learning Strategy	Assessment Strategy	Alignment to CLO
7.	Parallel Operation of two Alternators Effect of Unequal Voltages on Distribution of Load Time Period of Oscillation, Maximum Power Output	Lecture, Multimedia, Group Discussion	Feedback, Q&A	CLO 4
8.	Mathematical Problems on Alternator	Lecture, Multimedia	Feedback, Q&A Quiz#2	CLO 4
9.	General Principle of Operation, Method of Starting of Synchronous Motor,Motor on Load with Constant Excitation, Power Flow within a Synchronous Motor, Equivalent Circuit of a Synchronous Motor, Power Developed by a Synchronous Motor	Lecture, Multimedia, Practical Example	Feedback, Q&A , Assignment	CLO 1 CLO 2

Course Plan Mapped with CLO

Week No.	Topics	Teaching-Learning Strategy	Assessment Strategy	Alignment to CLO
10.	Synchronous Motor with Different Excitations, Effect of increased Load with Constant Excitation, Effect of Changing Excitation of Constant Load, Different Torques of a Synchronous Motor, Power Developed by a Synchronous Motor, Alternative Expression for Power Developed	Lecture, Multimedia, Group Discussion	Feedback, Q&A	CLO 2 CLO 3 CLO 4
11.	Construction of V curves Hunting or Surging or Phase Swinging, Methods of Starting, Procedure for Starting a Synchronous Motor, Comparison between Synchronous and Induction Motors, Synchronous Motor Applications	Lecture, Multimedia, Practical Example	Feedback, Q&A, Quiz #3	CLO 3 CLO 4
12.	Mathematical Problems on Synchronous Motor	Lecture, Multimedia	Feedback, Q&A	CLO 4

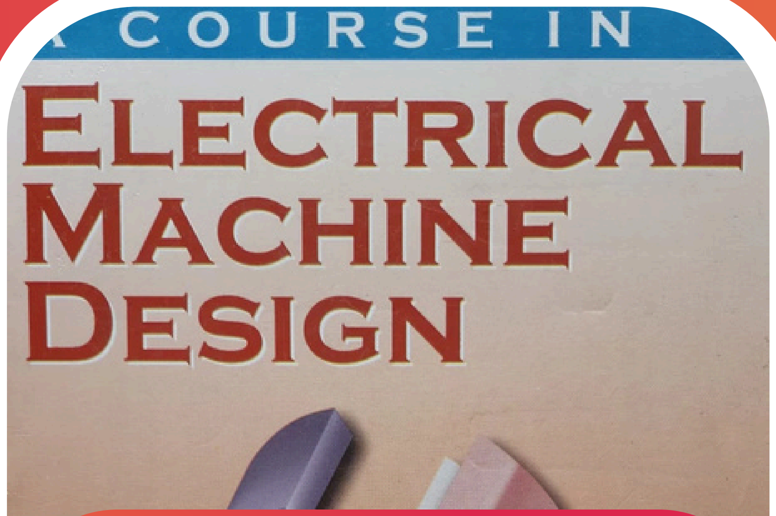
Course Plan Mapped with CLO

Week No.	Topics	Teaching-Learning Strategy	Assessment Strategy	Alignment to CLO
13.	Working principle of Induction Motor,Construction of Induction Motor, Squirrel-cage Rotor, Phase-wound Rotor	Lecture, Multimedia,	Feedback, Q&A , Assignment	CLO 1 CLO 2
14.	Production of Rotating Field, Rotation of Rotor, Slip, Percentage of Slip, Analysis of the Relation between Torque and Rotor Power Factor	Lecture, Multimedia,	Feedback, Q&A	CLO 2 CLO 3
15.	Starting Torque, Condition for Maximum Starting Torque, Effect of Change in Supply Voltage on Starting Torque, Torque under Running Condition	Lecture, Multimedia,	Feedback, Q&A	CLO 4
16.	Analysis of the Relation between Torque and Slip & Torque and Speed, Full-load Torque and Maximum Torque, Starting Torque and Maximum Torque	Lecture, Multimedia,	Feedback, Q&A , Quiz #4	CLO 4

Course Plan Mapped with CLO

Week No.	Topics	Teaching-Learning Strategy	Assessment Strategy	Alignment to CLO
17.	Torque/Speed Curve, Shape of Torque/ Speed Curve, Torque/Speed Characteristic, Equivalent Circuit of an Induction Motor, Induction Motor Operating as a Generator, Power Stages in an Induction Motor, Torque Developed by an Induction Motor, Torque, Mechanical Power and Rotor Output	Lecture, Multimedia ,Group Discussion	Feedback, Q&A , Assignment	CLO 3 CLO 4
18.	Mathematical Problem on Induction Motor	Lecture, Multimedia	Feedback, Q&A	CLO 4

Reference Books



Performance & Design of AC machines

M.G. Say

Electrical Machinery

P.S. Bhimbra

A Course in Electrical Machine Design

A.K. Sawhney

WEEK 01

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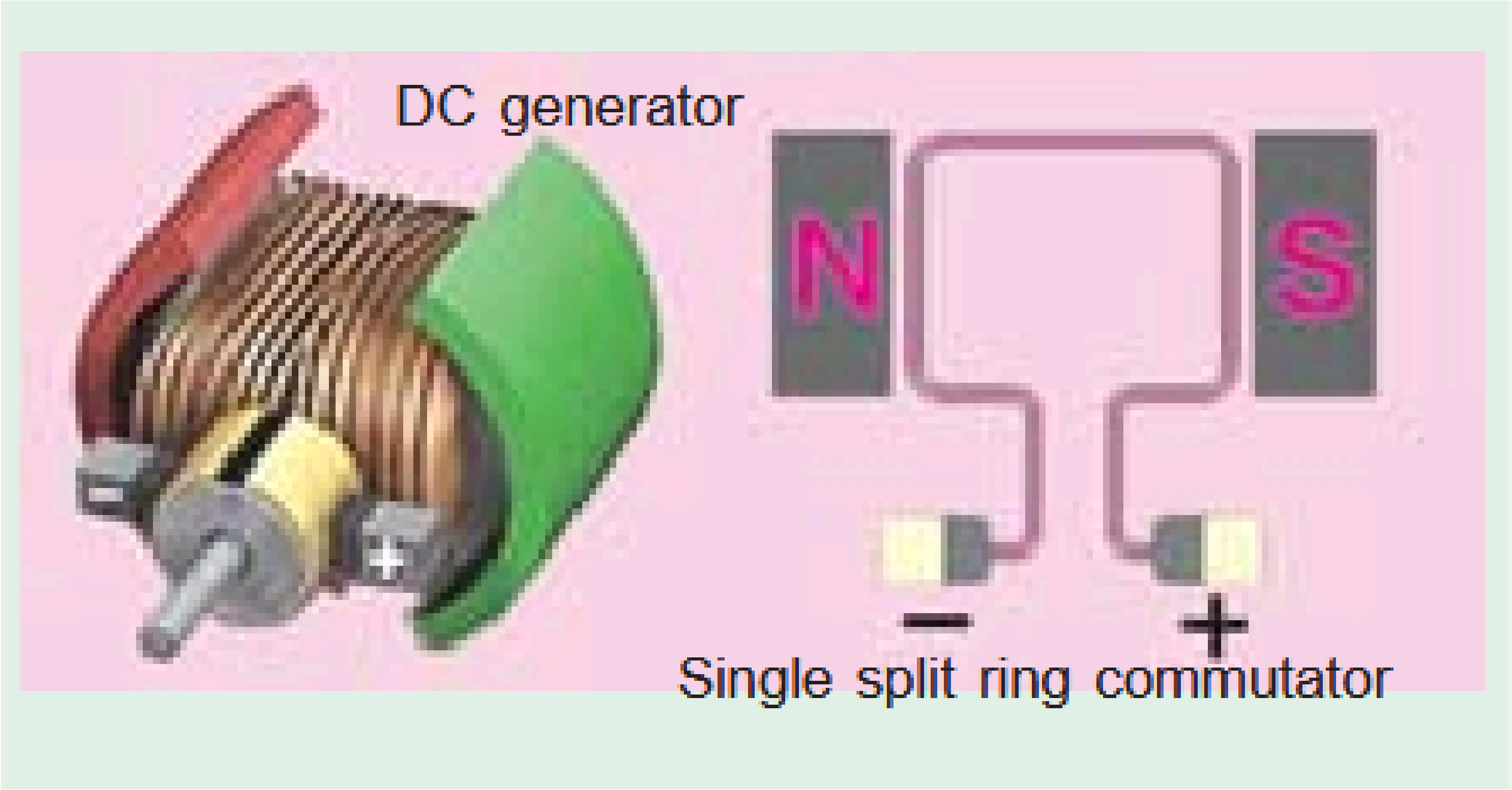
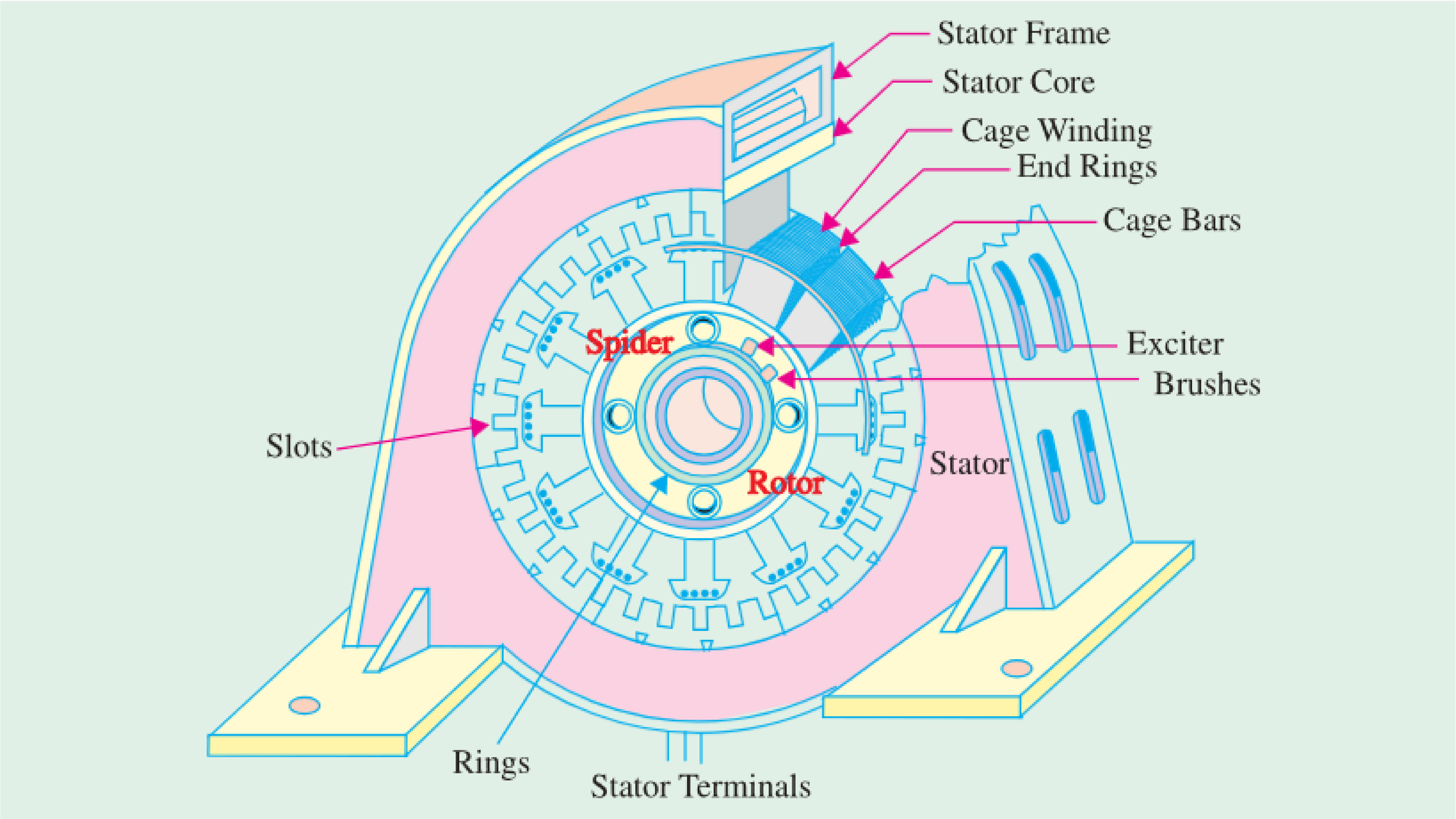
Alternators

A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as d.c. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in d.c. generators, the ***armature rotates*** and the field system is ***stationary***, the arrangement in alternators is just the reverse of it. In their case, standard construction consists of armature winding mounted on a stationary element called ***stator*** and field windings on a rotating element called rotor.



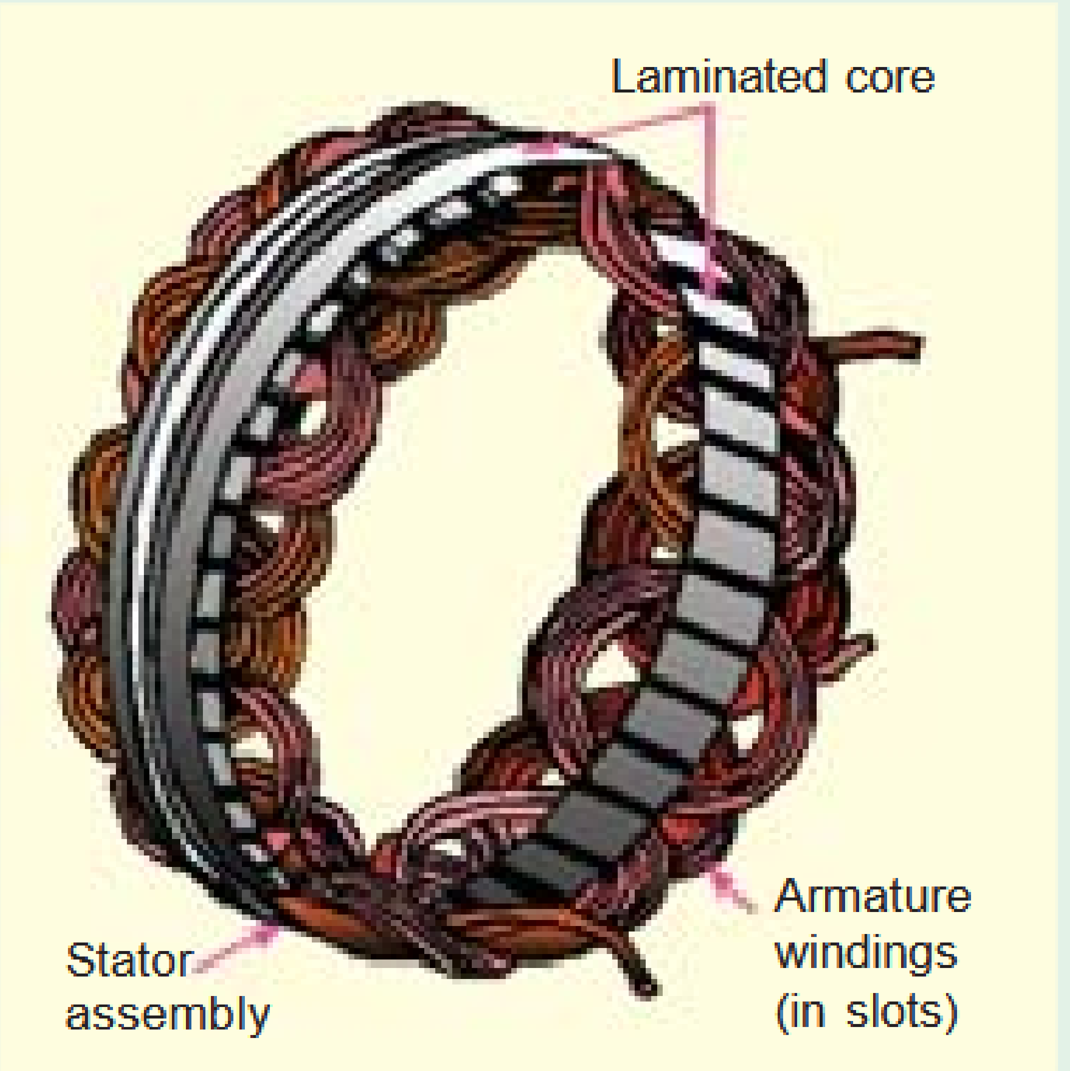
Alternators



Stationary Armature

Advantages of having stationary armature (and a rotating field system) are :

1. The output current can be led directly from fixed terminals on the stator (or armature windings) to the load circuit, without having to pass it through brush-contacts.

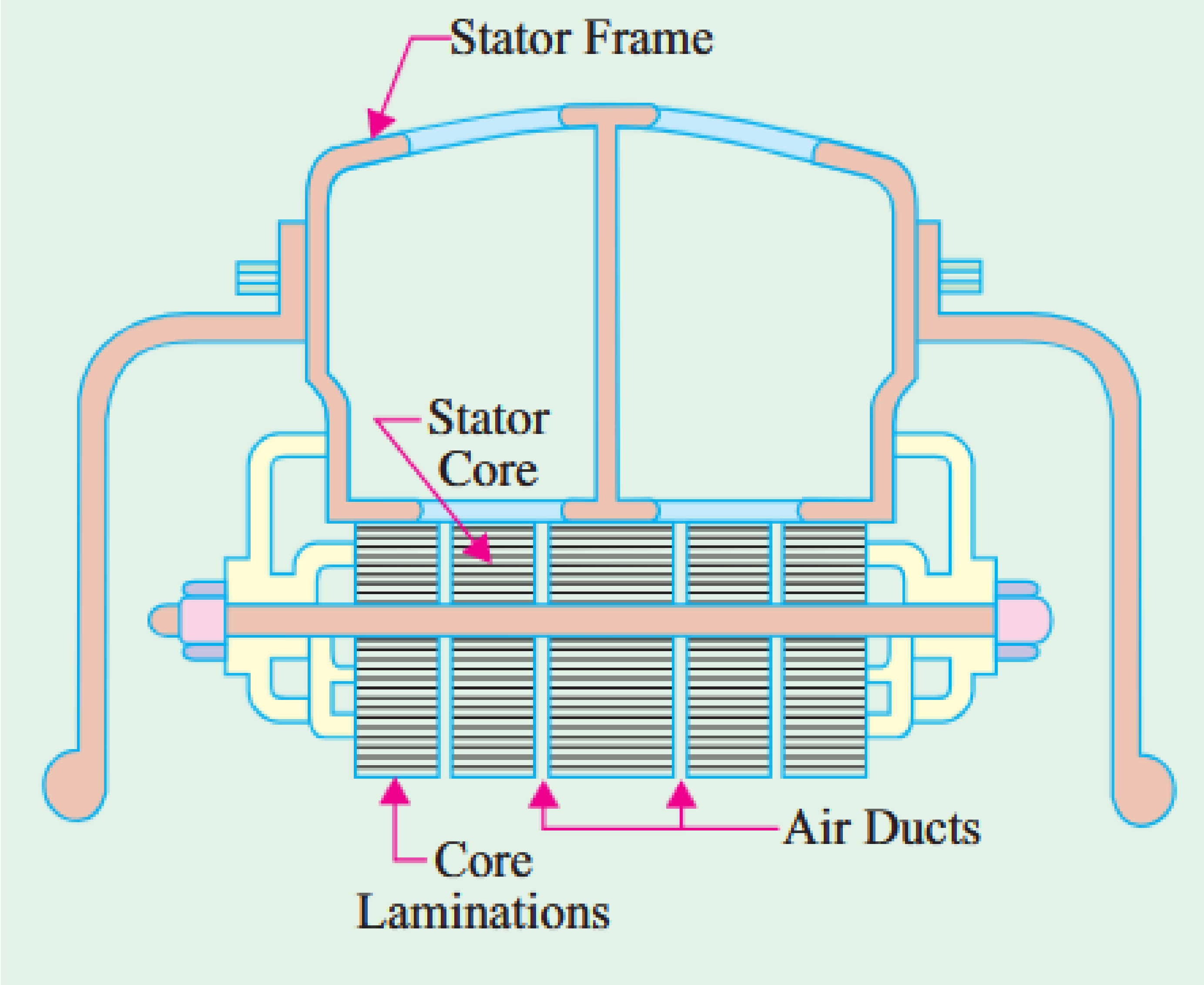


Stationary armature windings

2. It is easier to insulate stationary armature winding for high a.c. voltages, which may have as high a value as 30 kV or more.
3. The sliding contacts *i.e.* slip-rings are transferred to the low-voltage, low-power d.c. field circuit which can, therefore, be easily insulated.
4. The armature windings can be more easily braced to prevent any deformation, which could be produced by the mechanical stresses set up as a result of short-circuit current and the high centrifugal forces brought into play.

Construction

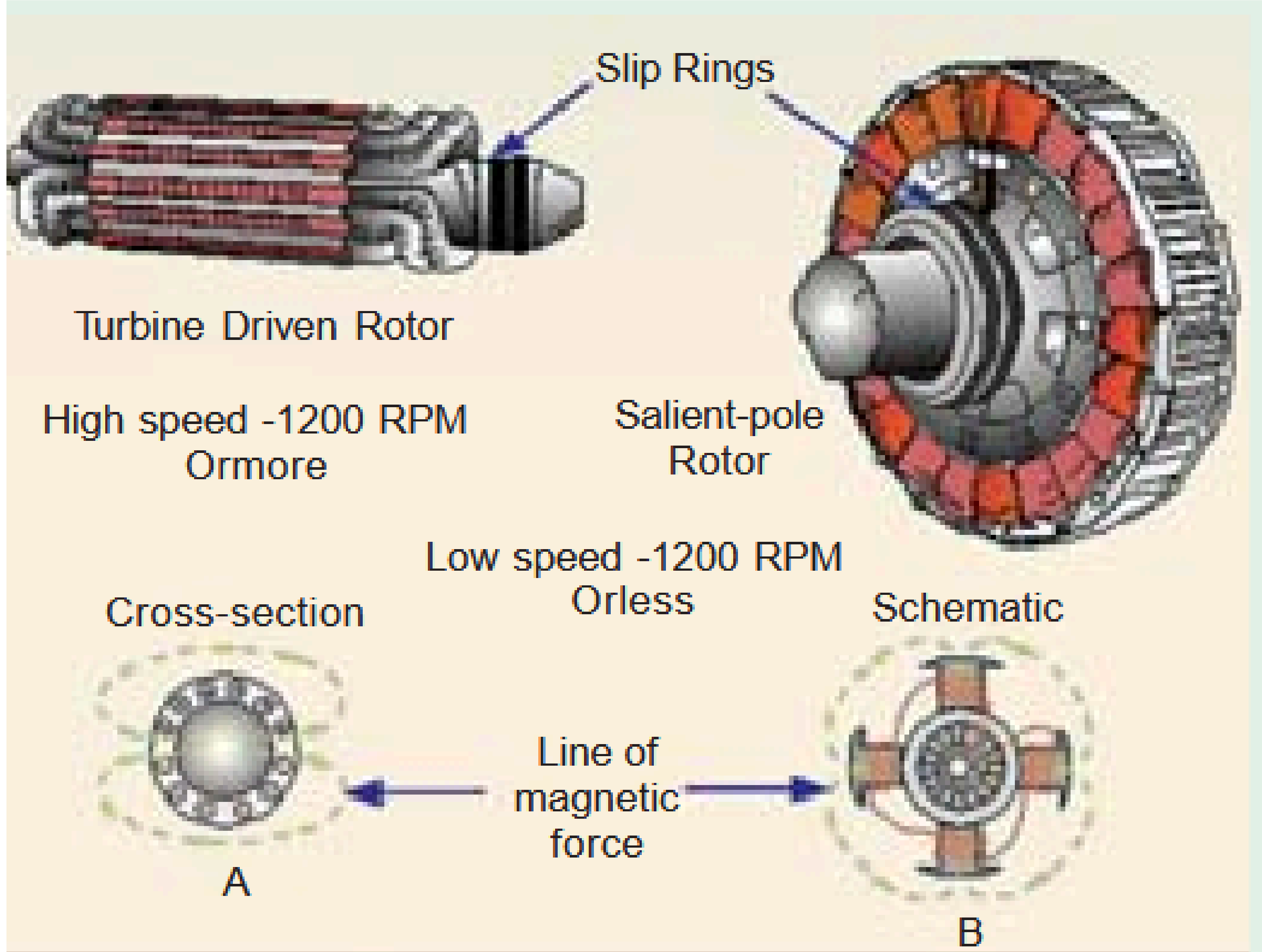
Stator Frame



Streetbike stator

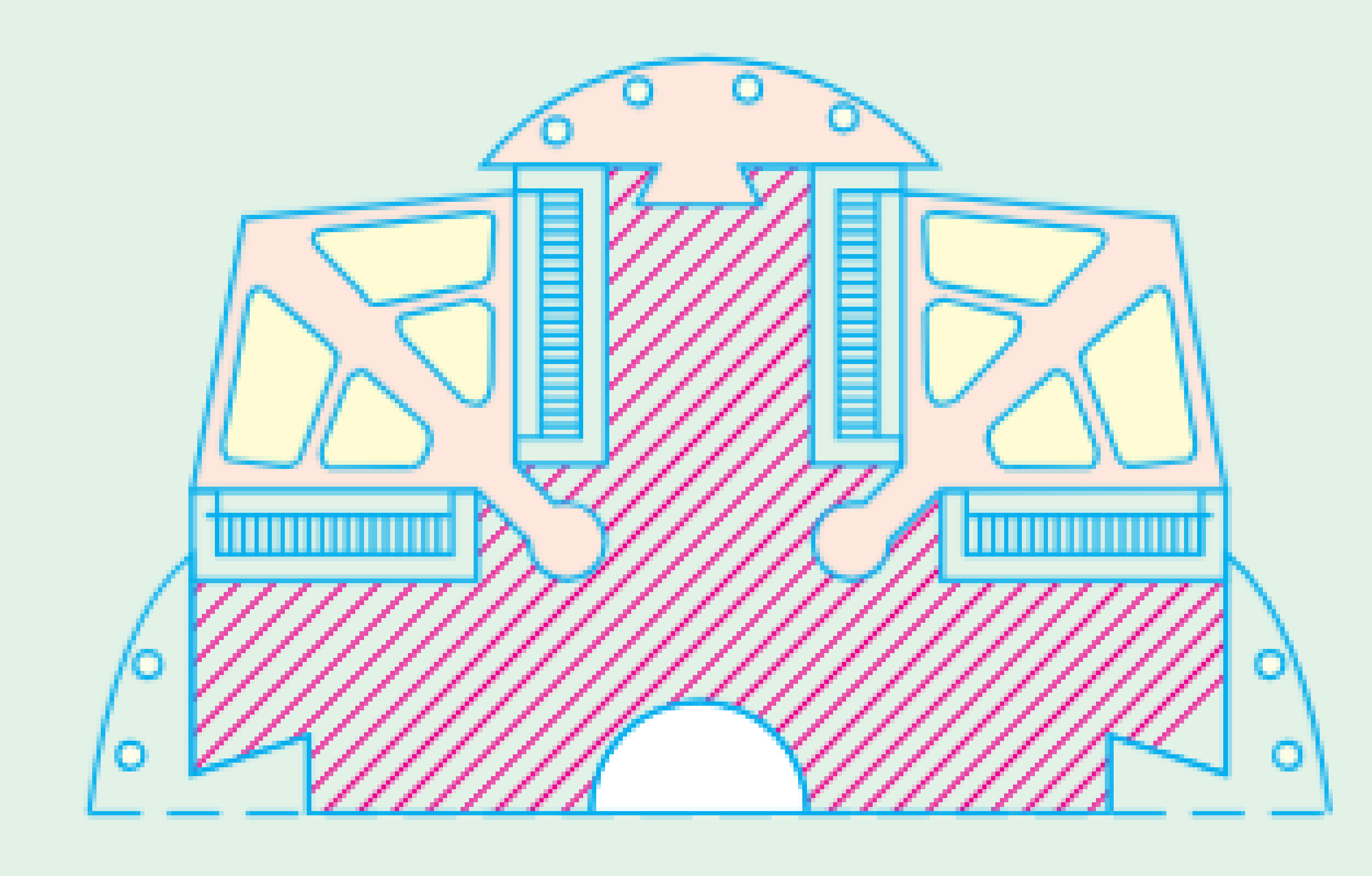
Rotor

(i) Salient (or projecting) Pole Type

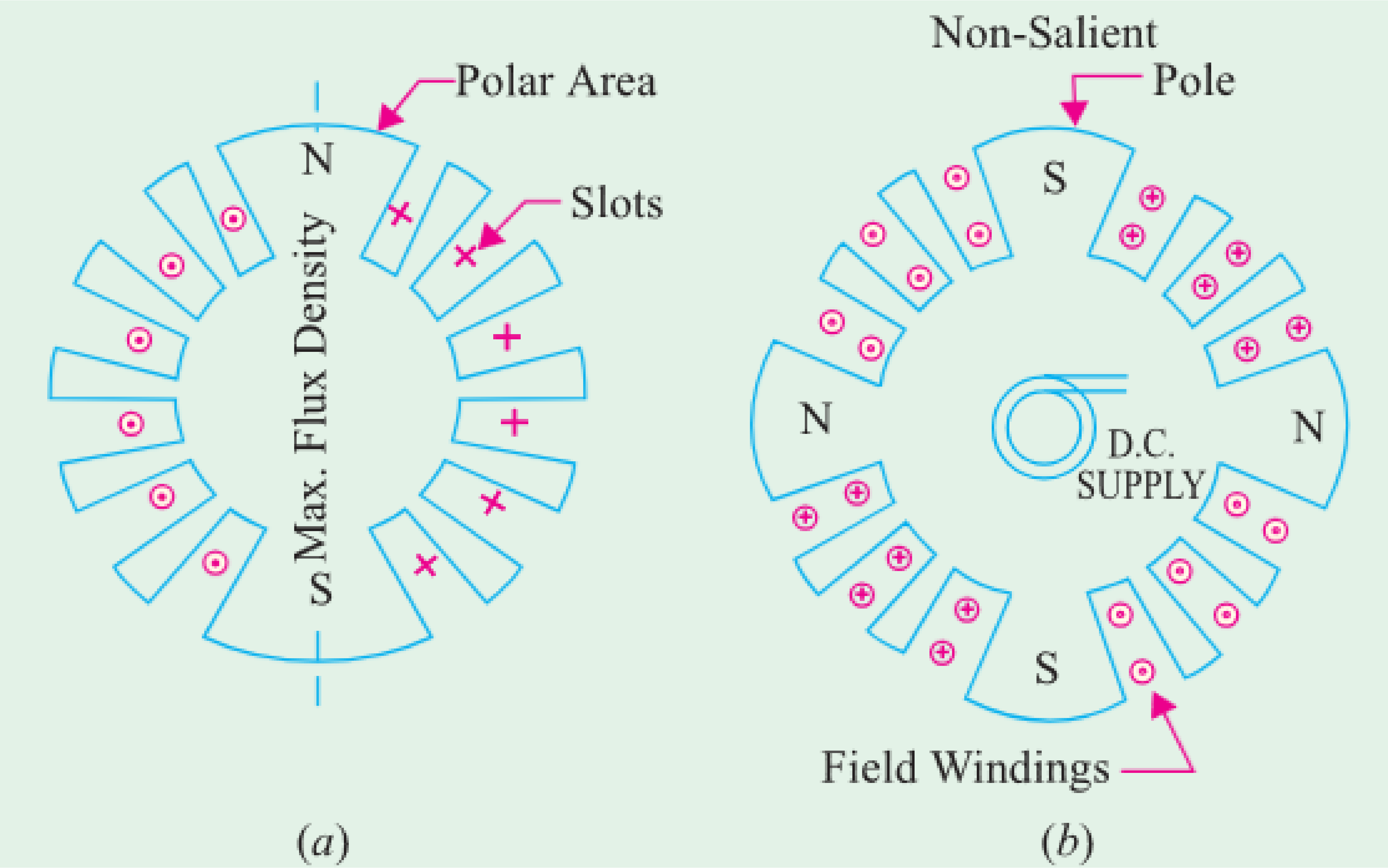


Types of rotors used in alternators

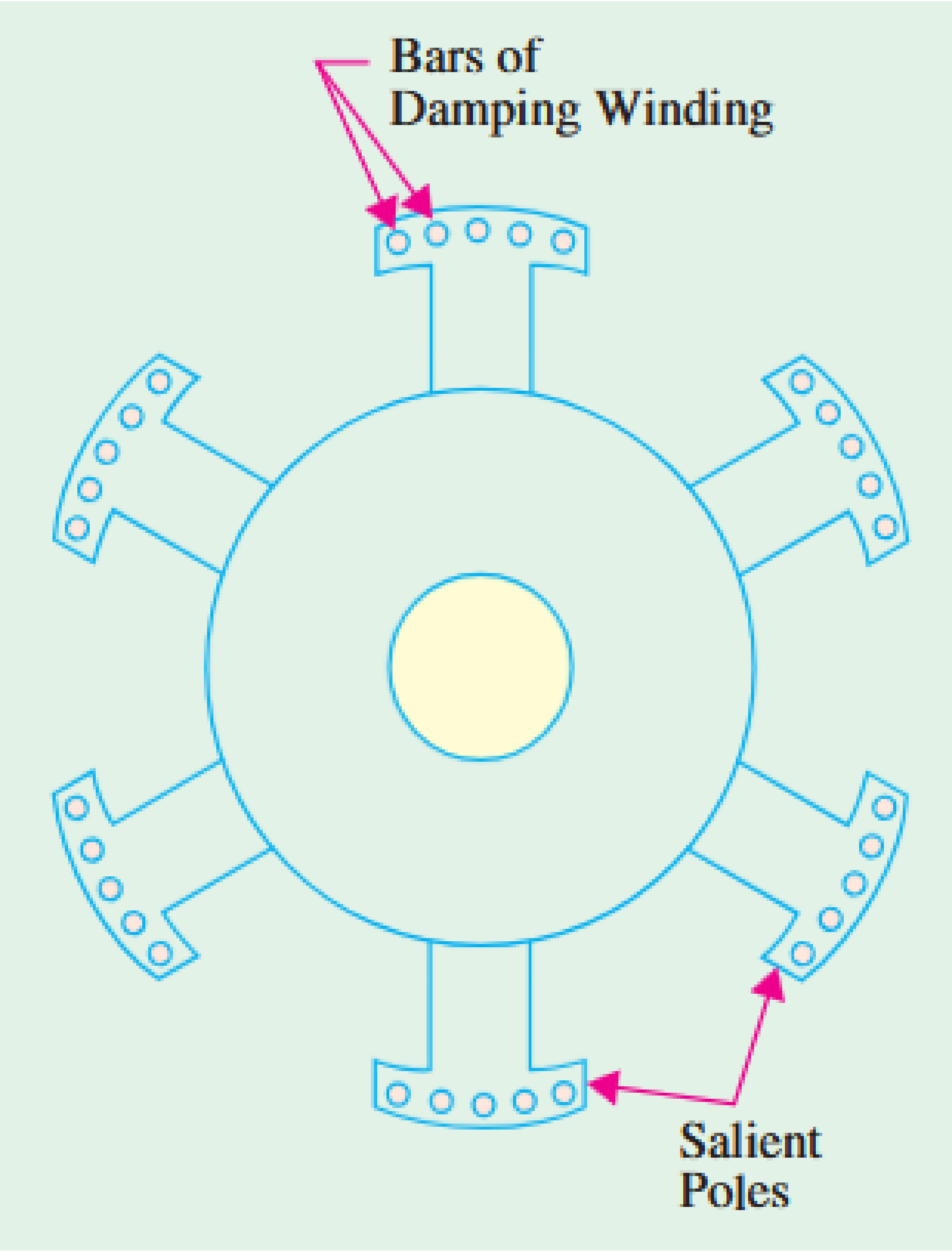
(ii) Smooth Cylindrical Type

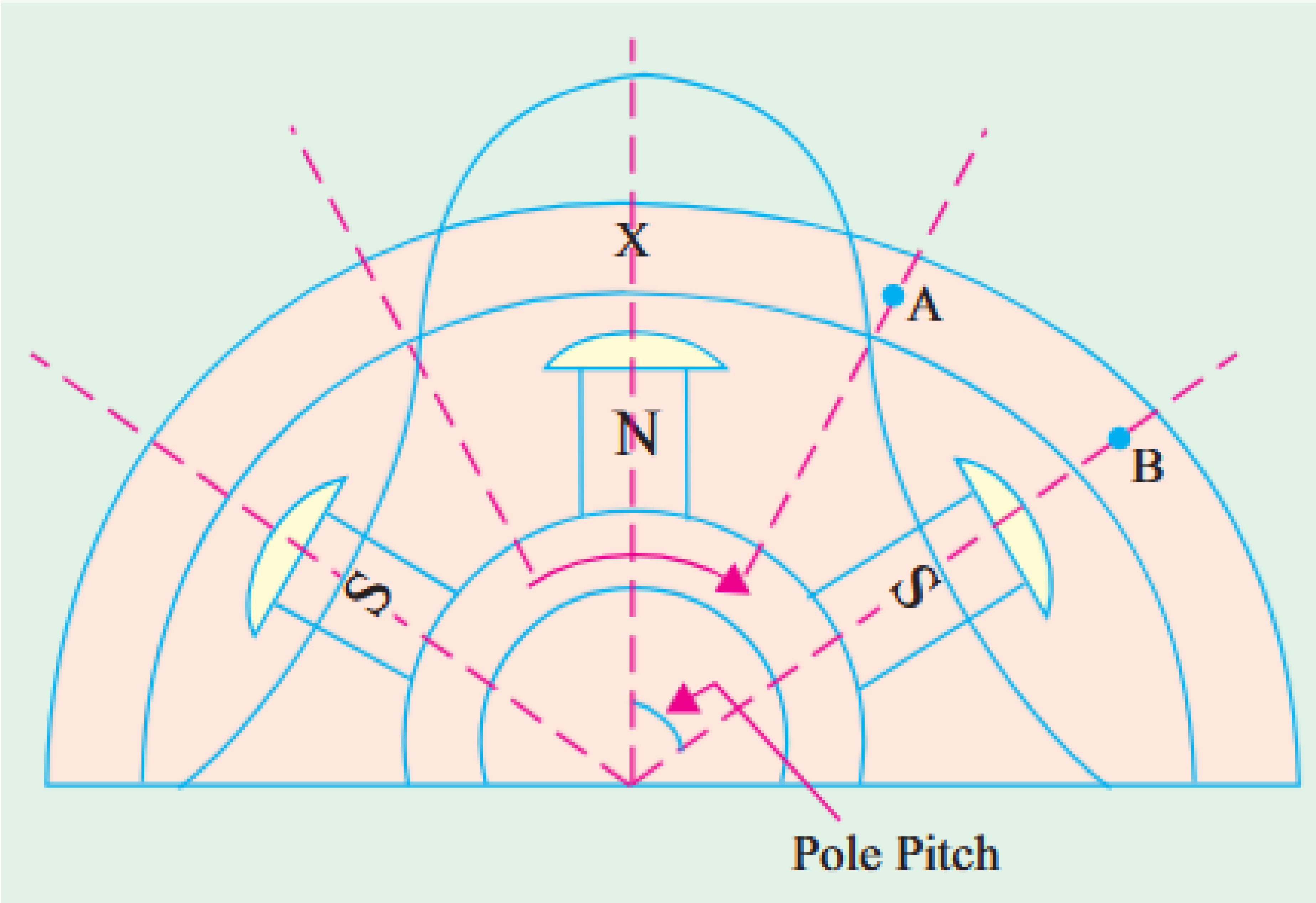


Damper Windings



Speed and Frequency





Speed and Frequency

Since one cycle of e.m.f. is produced when a pair of poles passes past a conductor, the number of cycles of e.m.f. produced in one revolution of the rotor is equal to the number of pair of poles.

$\therefore$ No. of cycles/revolution = $P/2$ and No. of revolutions/second = $N/60$

$$\therefore \quad \text{frequency} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ Hz}$$

or

$$f = \frac{PN}{120} \text{ Hz}$$

N is known as the synchronous speed, because it is the speed at which an alternator must run, in order to generate an e.m.f. of the required frequency. In fact, for a given frequency and given number of poles, the speed is fixed. For producing a frequency of 60 Hz, the alternator will have to run at the following speeds :

No. of poles	2	4	6	12	24	36
Speed (r.p.m.)	3600	1800	1200	600	300	200

WEEK 02

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Armature Windings

The two types of armature windings most commonly used for 3-phase alternators are :

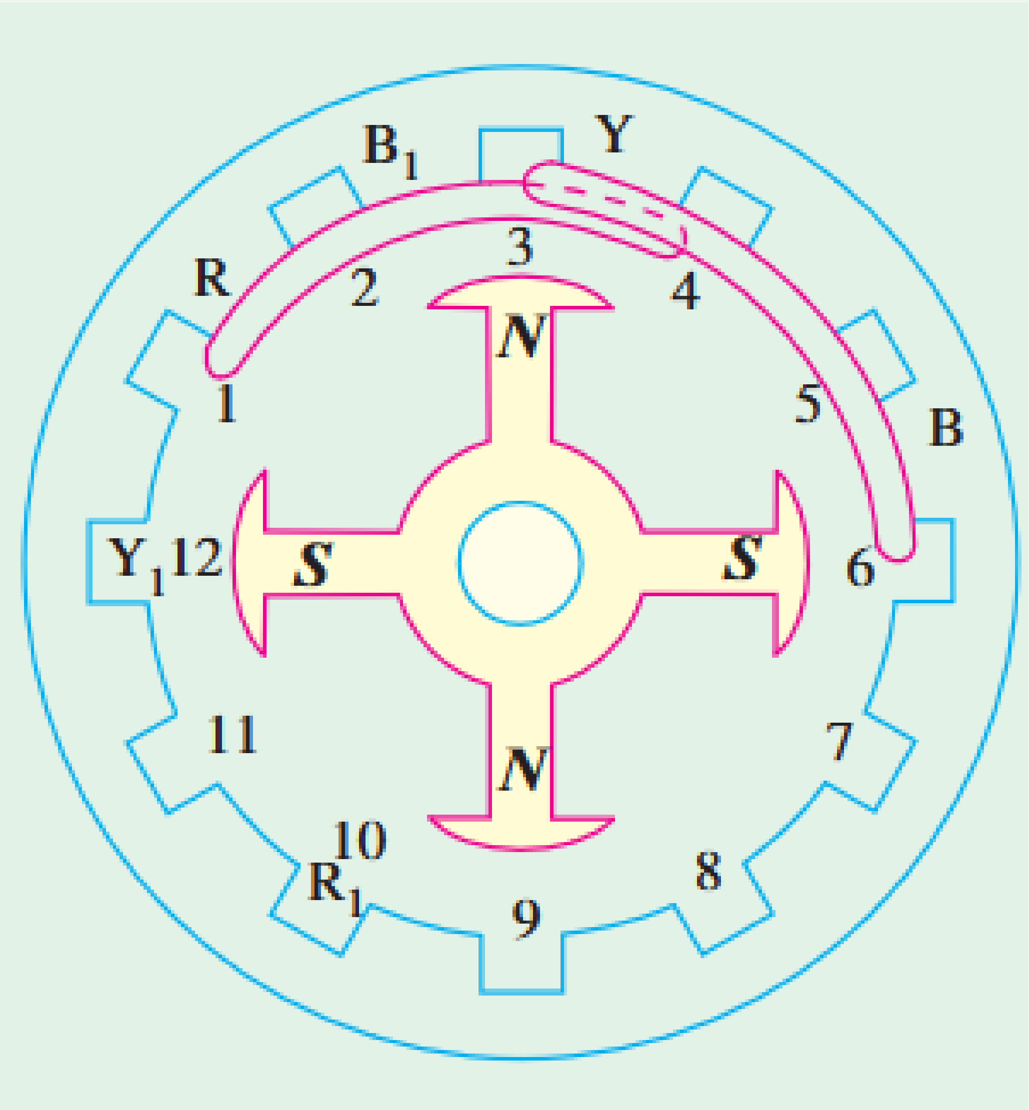
(i) *single-layer winding*

(ii) *double-layer winding*

Single-layer Winding

It is variously referred to as concentric or chain winding. Sometimes, it is of simple bar type or wave winding.

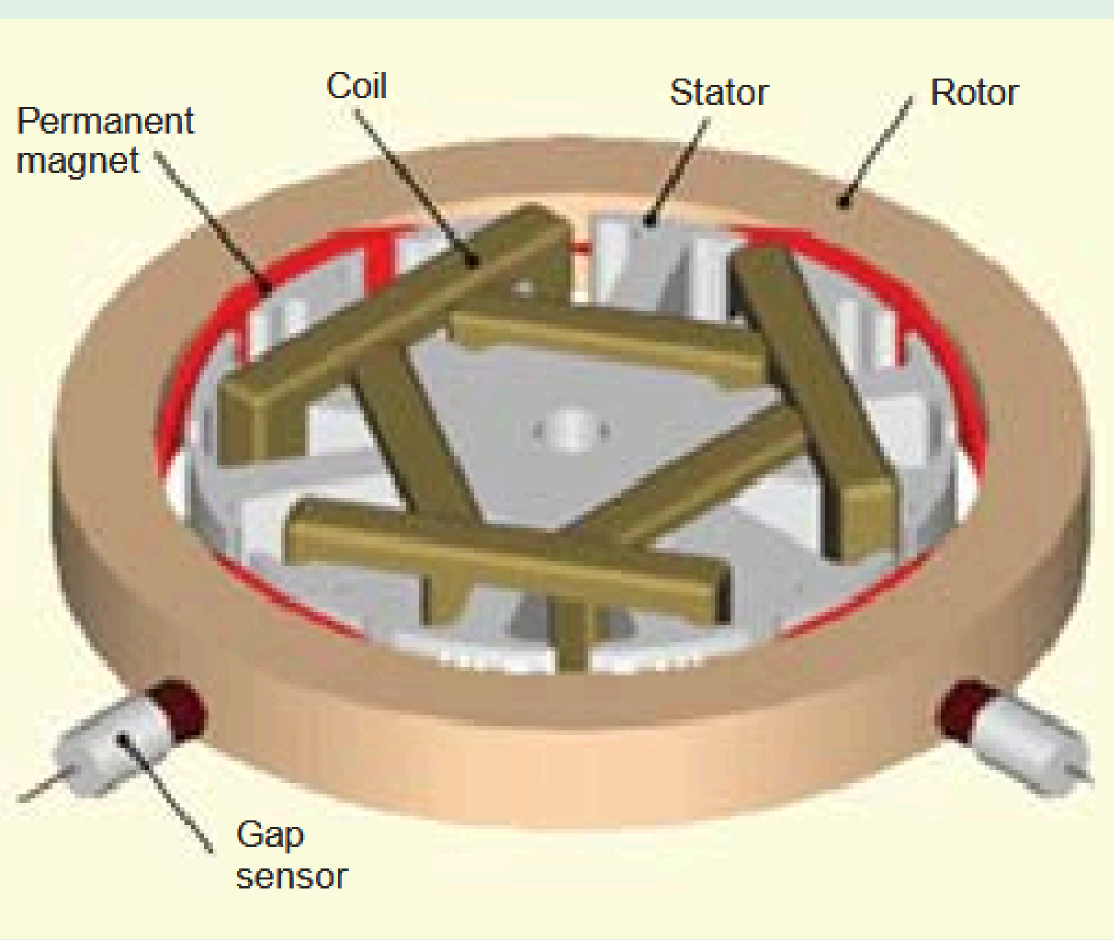
D.C. Armature



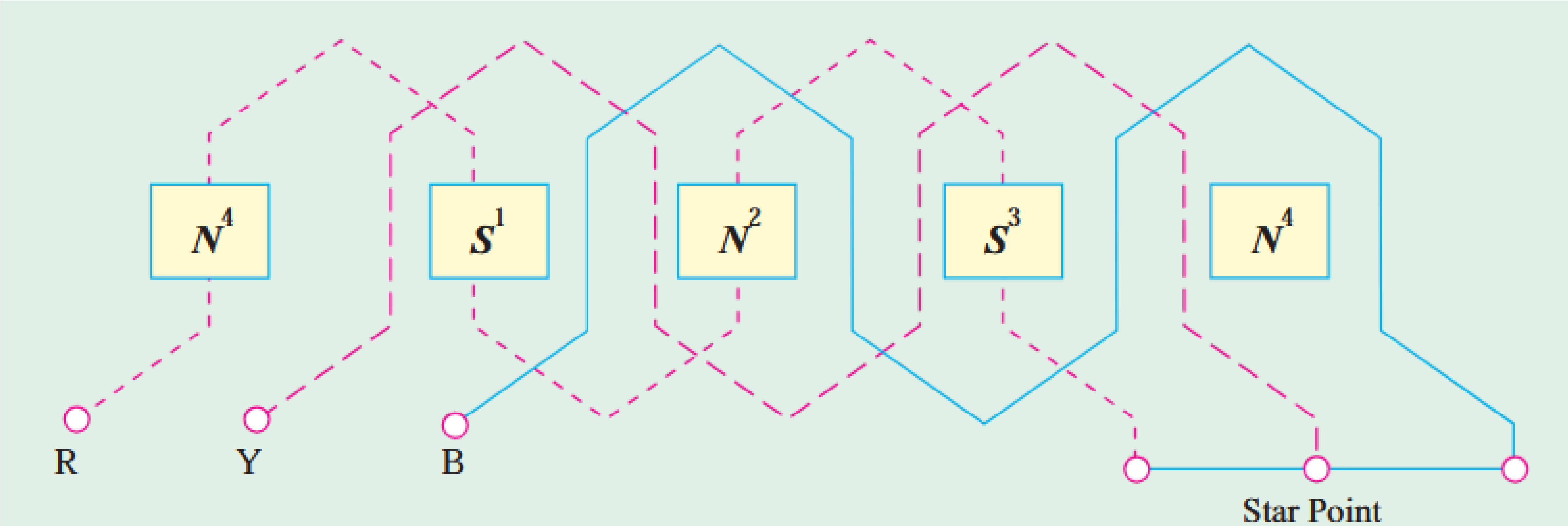
Single-layer Winding

Single Layer Winding in Armature Winding refers to an arrangement in which only one conductor or coil side is placed in each armature slot of the machine.

It is one of the simplest forms of armature windings used in electrical machines such as DC machines, alternators, and synchronous machines.



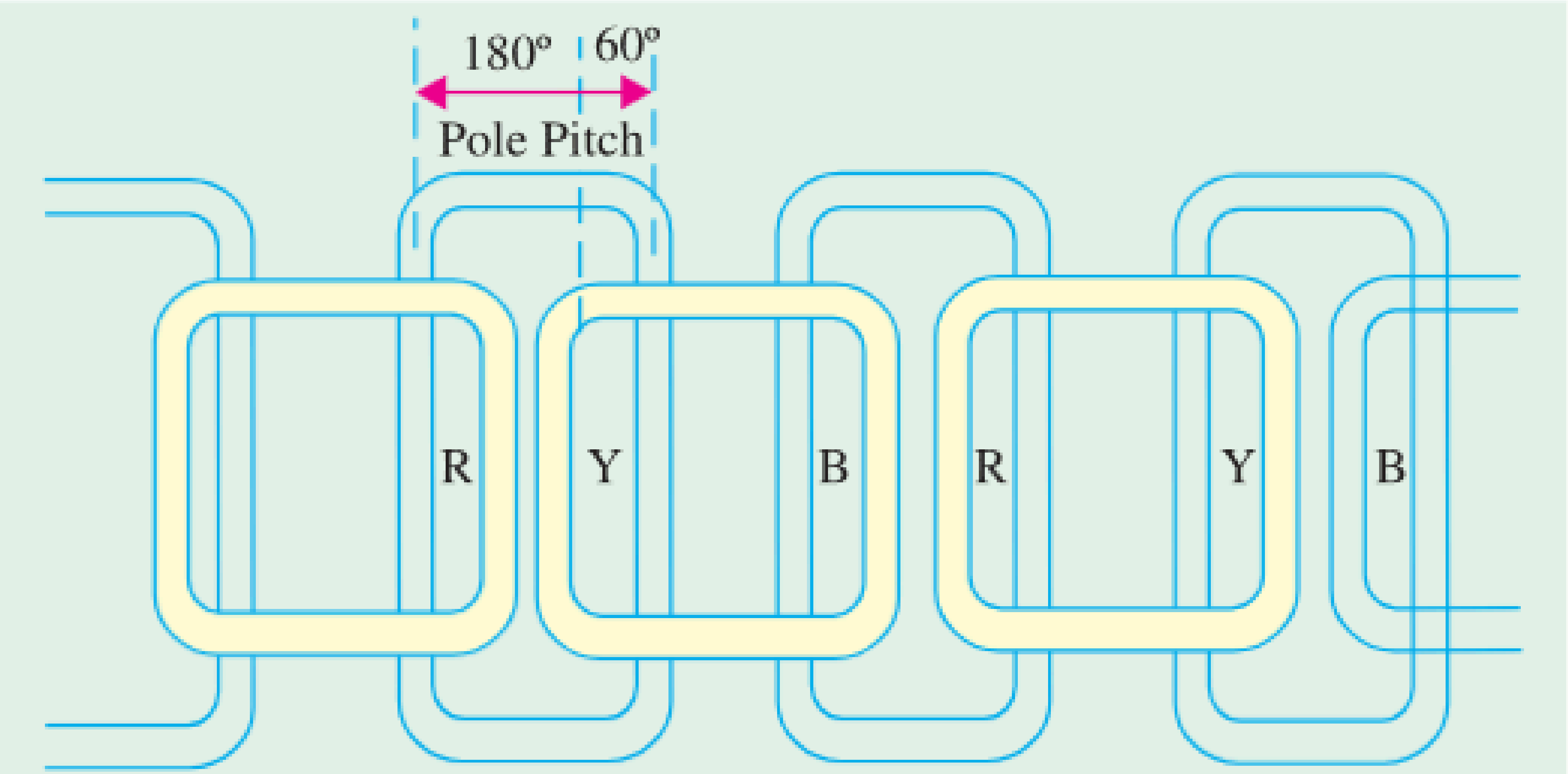
Single layer winding

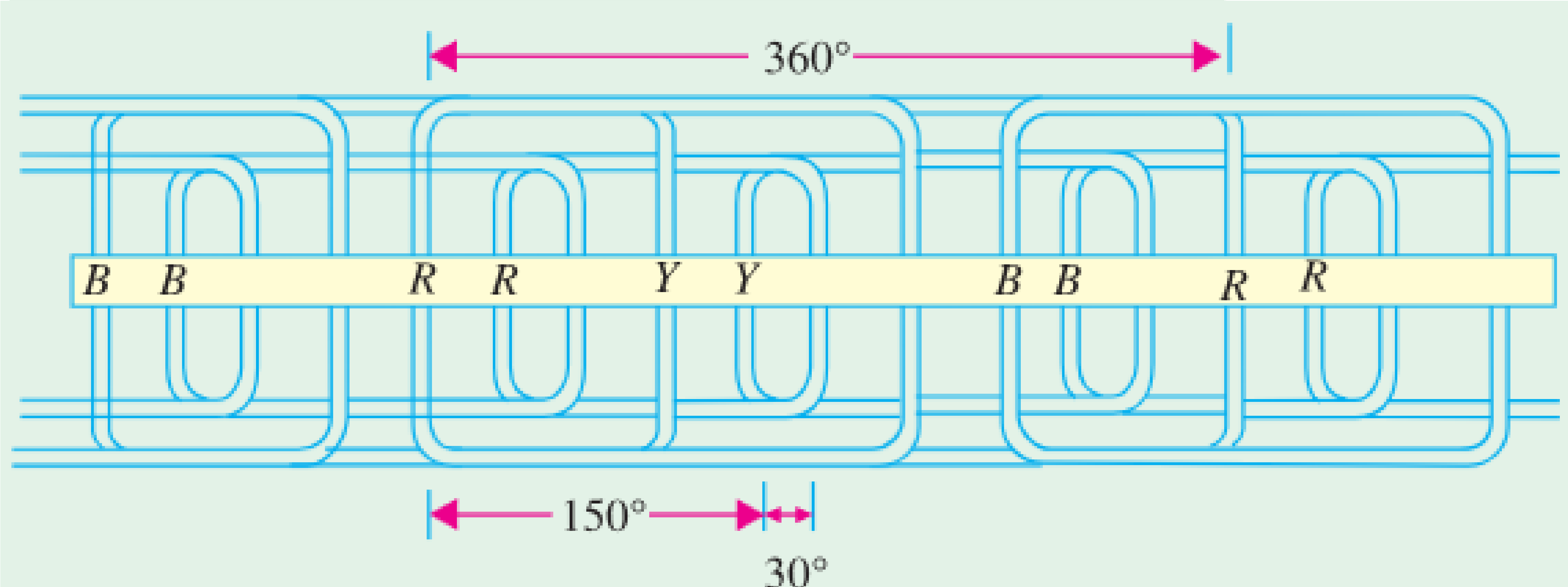


Concentric or Chain Windings

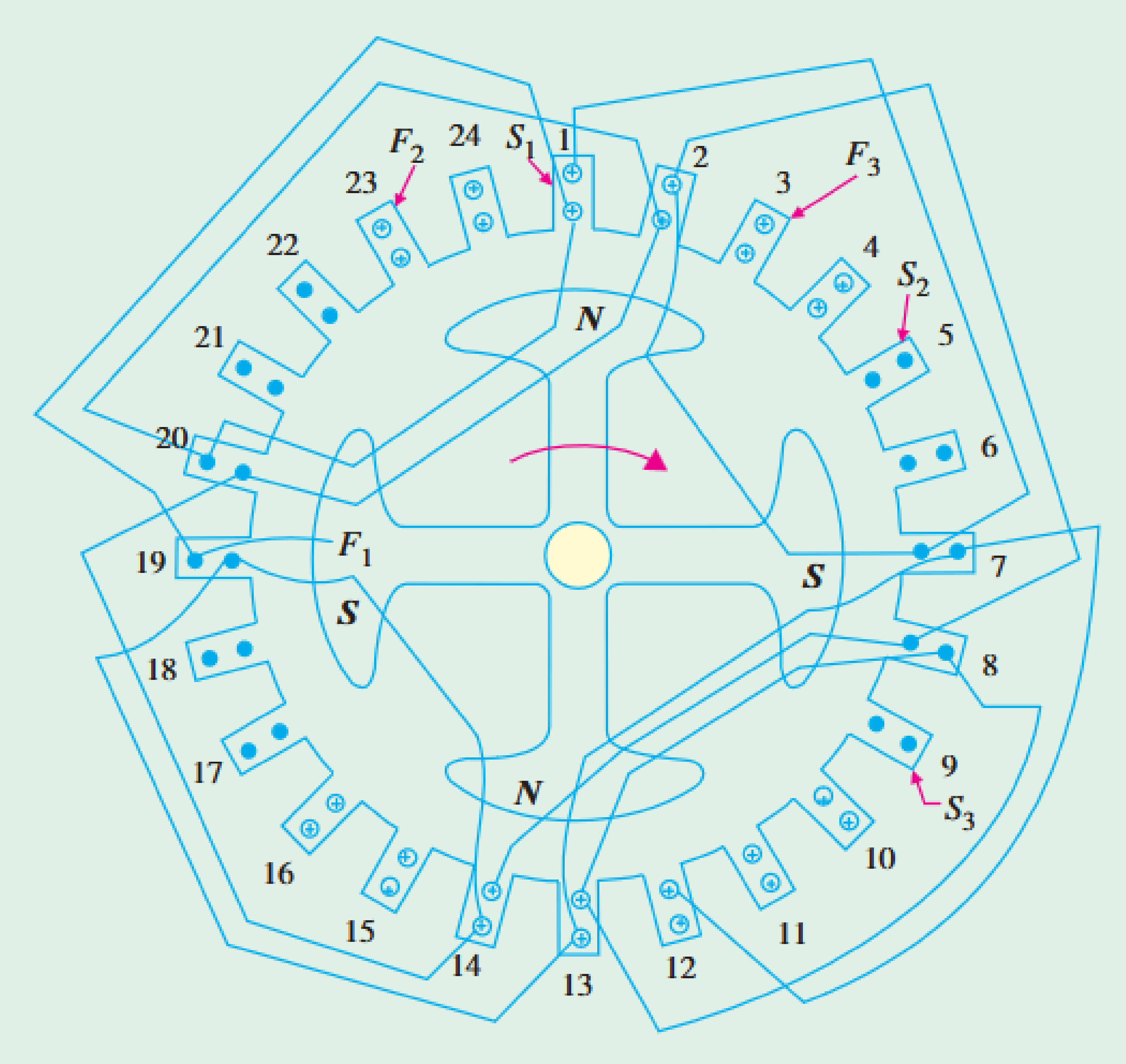
It would be noted that the polar group of each phase is 360° (electrical) apart in this type of winding

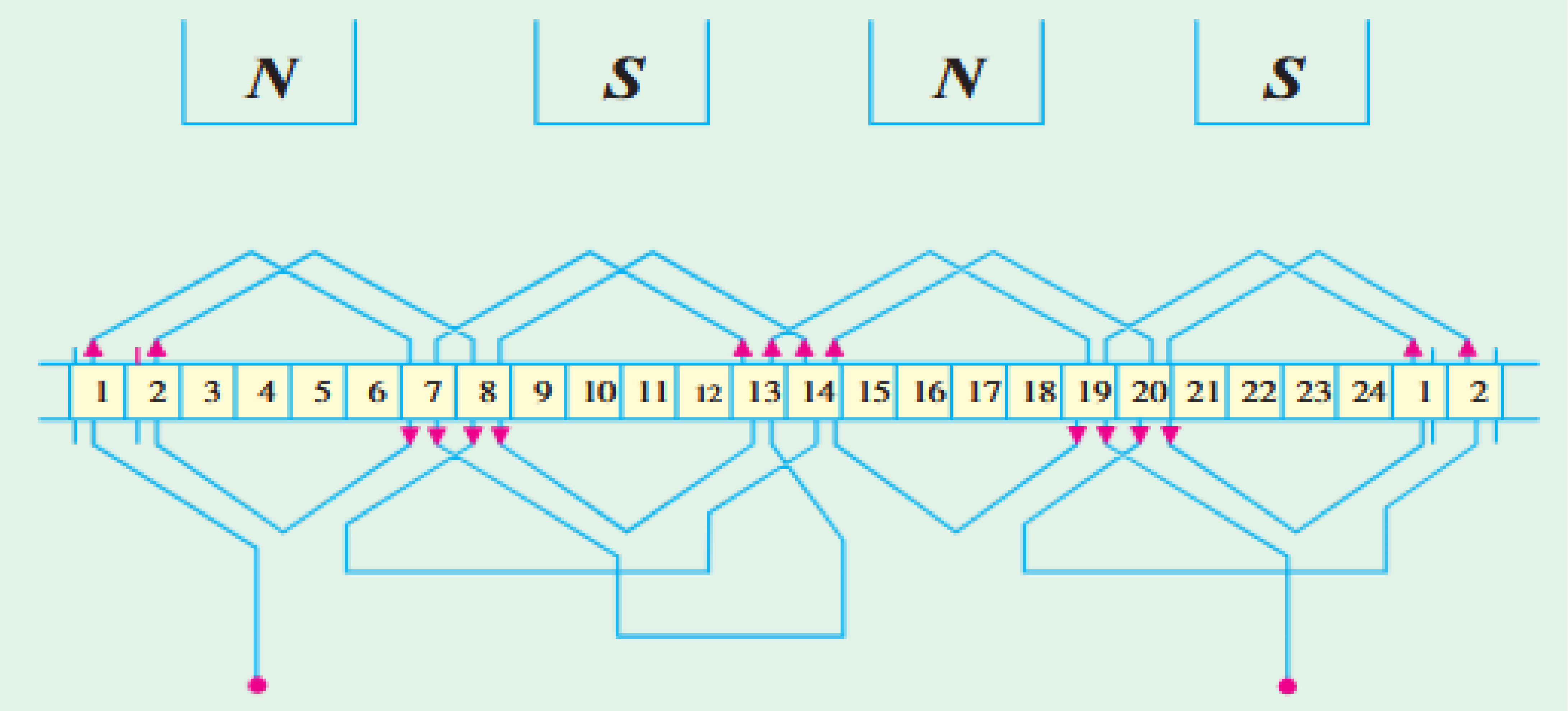
1. It is necessary to use two different shapes of coils to avoid fouling of end connections.
2. Since polar groups of each phase are 360 electrical degrees apart, all such groups are connected in the same direction.
3. The disadvantage is that short-pitched coils cannot be used.



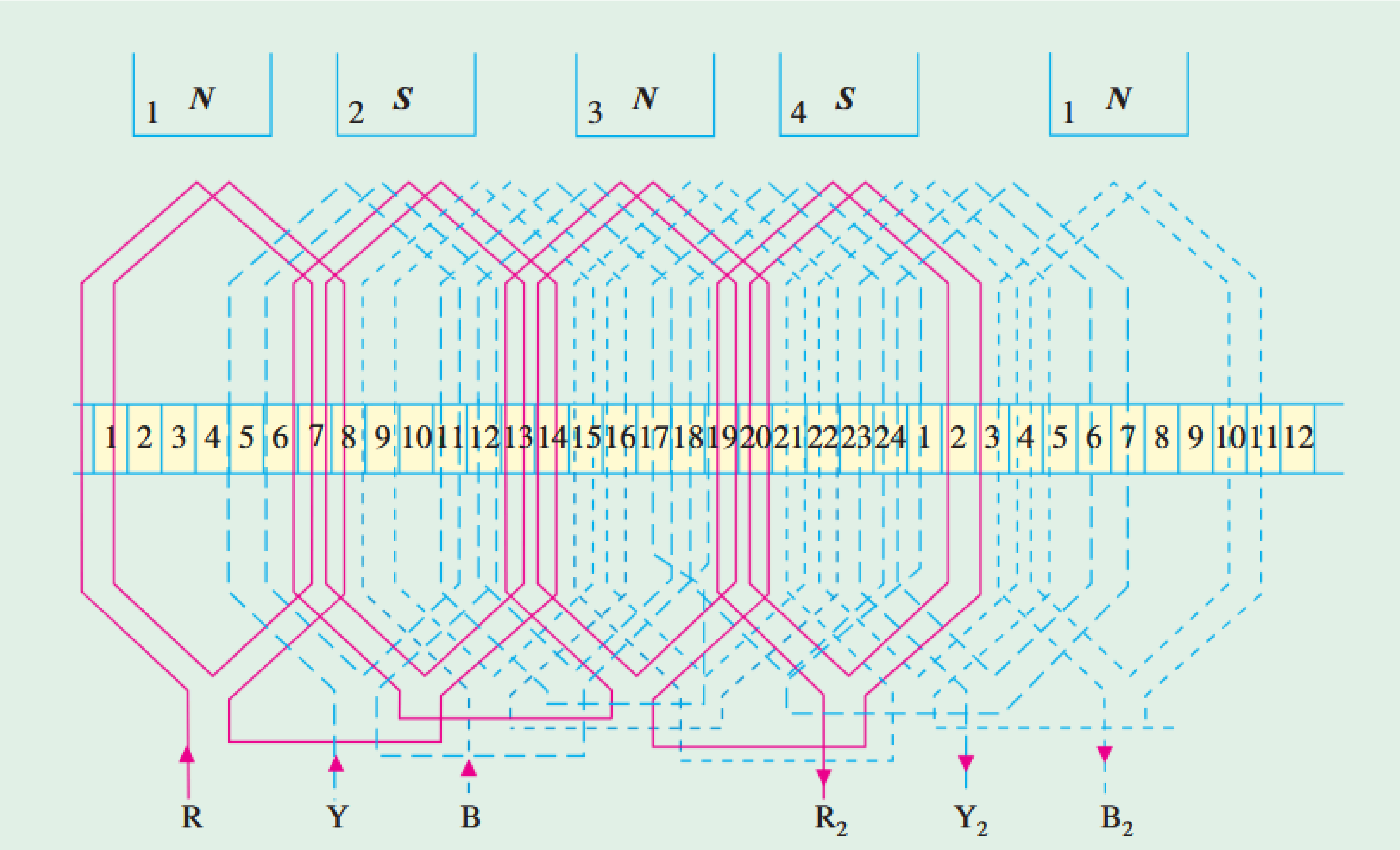


Two-Layer Winding

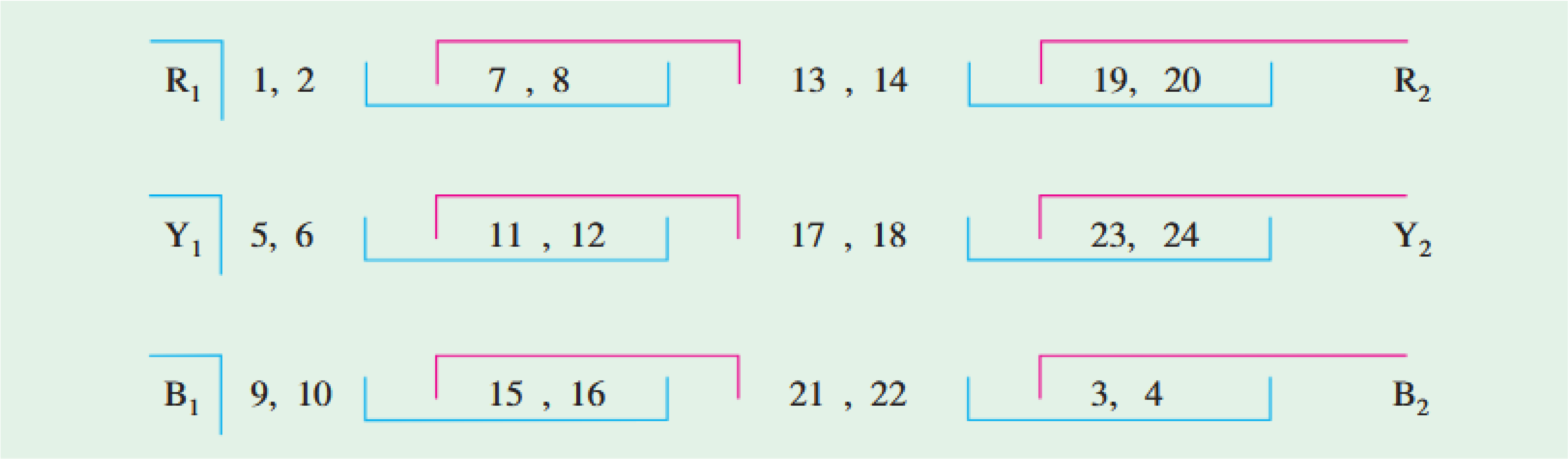




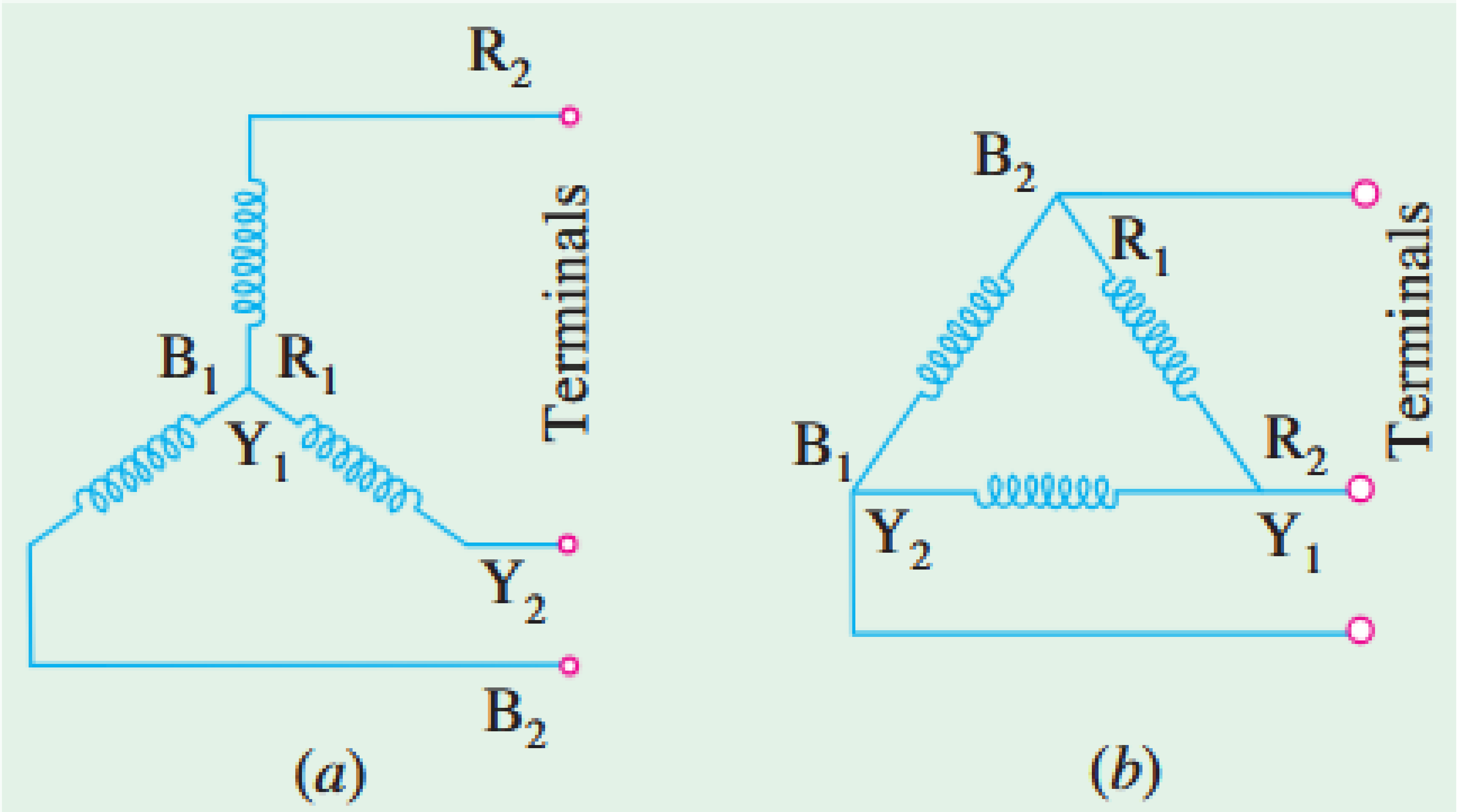
Concentric or Chain Windings

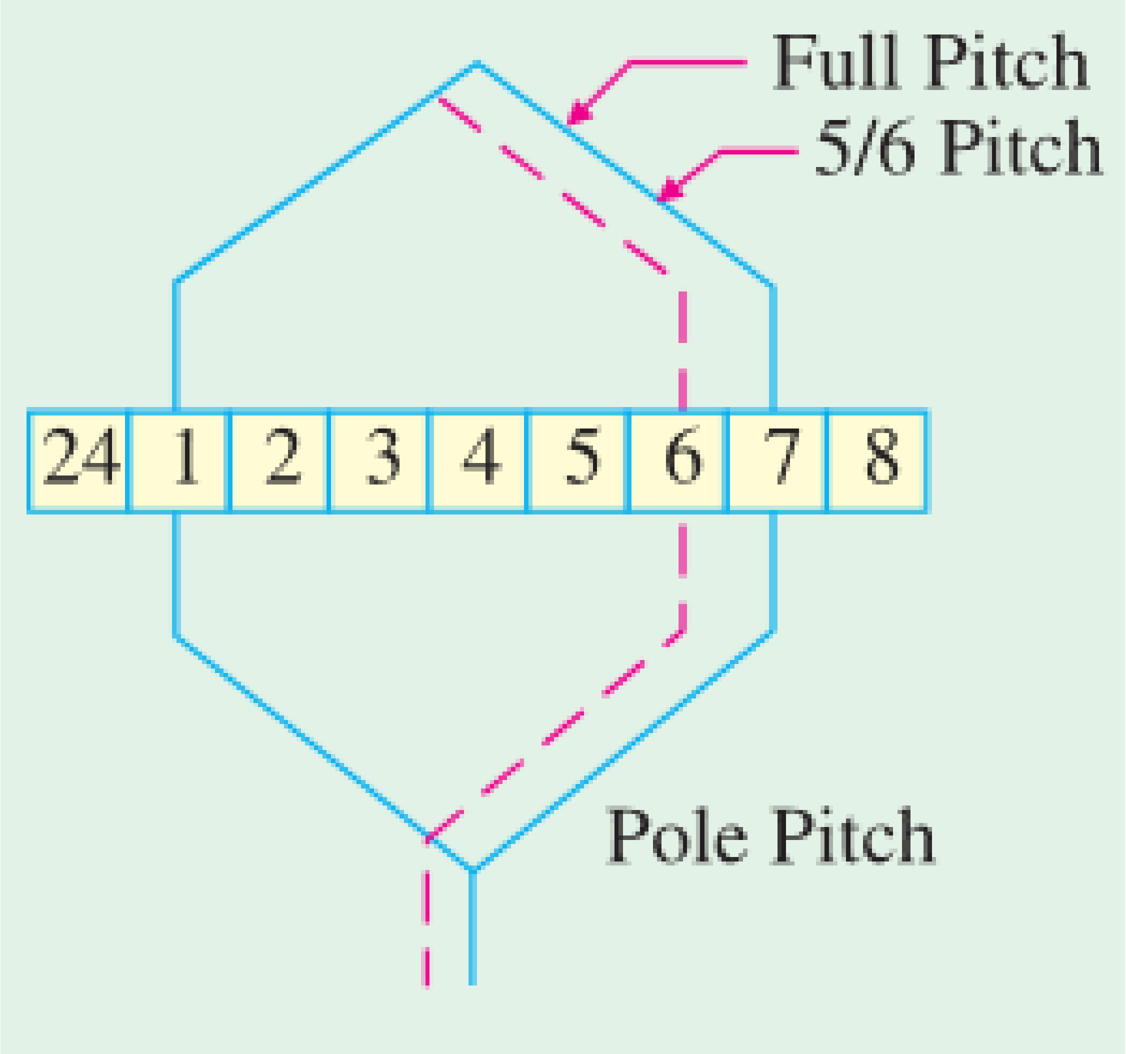


Concentric or Chain Windings



Wye and Delta Connections





Pitch factor/Chording factor

The pitch factor or coil-span factor k_p or k_c is defined as

$$= \frac{\text{vector sum of the induced e.m.fs. per coil}}{\text{arithmetic sum of the induced e.m.fs. per coil}}$$

It is always less than unity.

Let E_S be the induced e.m.f. in each side of the coil. If the coil were full-pitched *i.e.* if its two sides were one pole-pitch apart, then total induced e.m.f. in the coil would have been $= 2E_S$ [Fig. 37.17 (*a*).

If it is short-pitched by 30° (elect.) then as shown in Fig. 37.17 (*b*), their resultant is E which is the vector sum of two voltage 30° (electrical) apart.

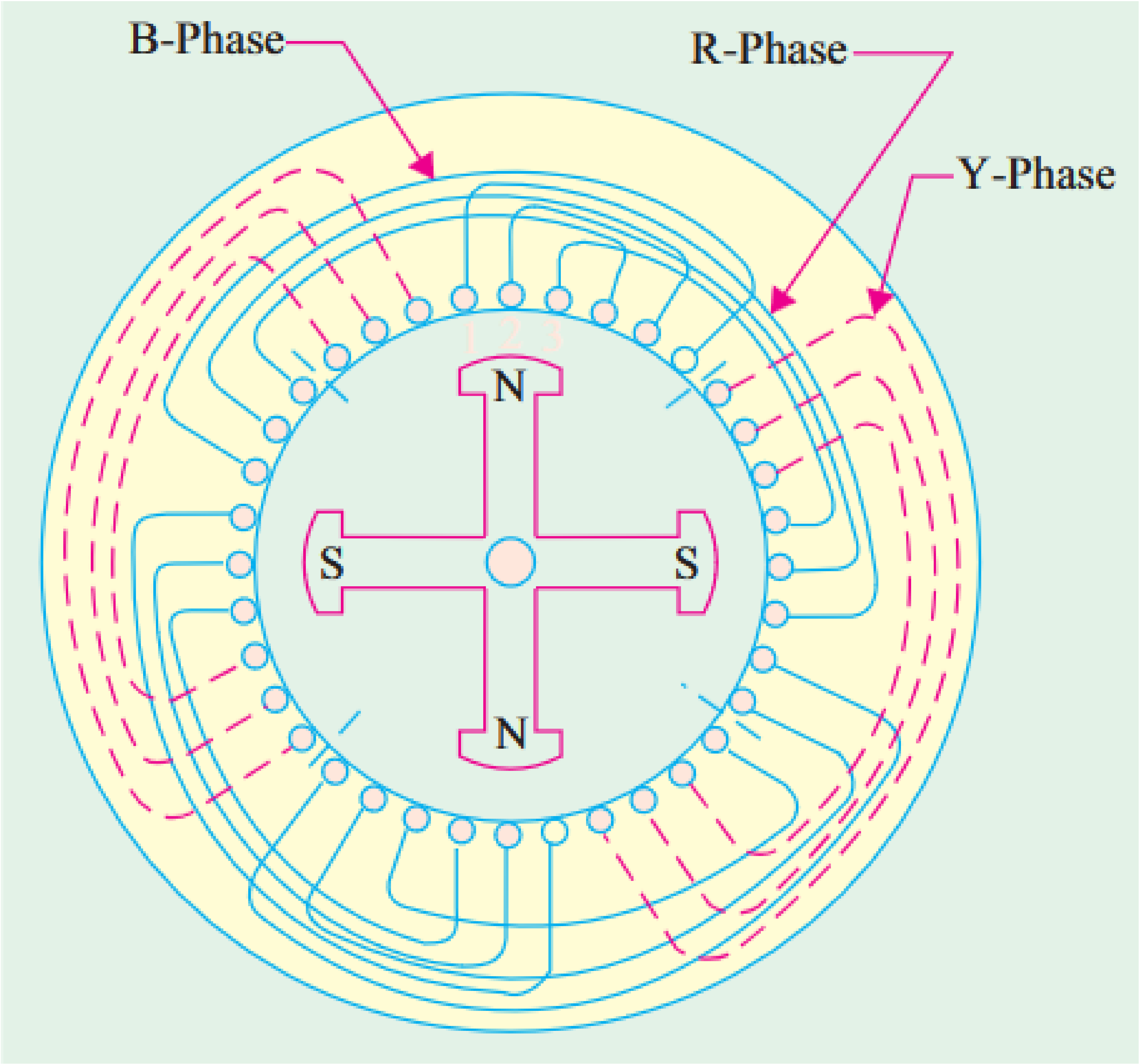
$$\therefore \qquad E = 2\,E_S \cos 30°/2 = 2E_S \cos 15°$$

$$k_c = \frac{\text{vector sum}}{\text{arithmetic sum}} = \frac{E}{2\,E_S} = \frac{2\,E_S \cos 15°}{2\,E_S} = \cos 15° = 0.966$$

Hence, pitch factor, $k_c = 0.966$.

Distribution or Breadth Factor or Winding Factor or Spread Factor

3-phase single-layer winding for a 4-pole alternator. It has a total of 36 slots i.e. 9 slots/pole. Obviously, there are 3 slots / phase / pole. For example, coils 1, 2 and 3 belong to R phase. Now, these three coils which constitute one polar group are not bunched in one slot but in three different slots. Angular displacement between any two adjacent slots = 180°/9 = 20° (elect.)



Distribution or Breadth Factor or Winding Factor or Spread Factor

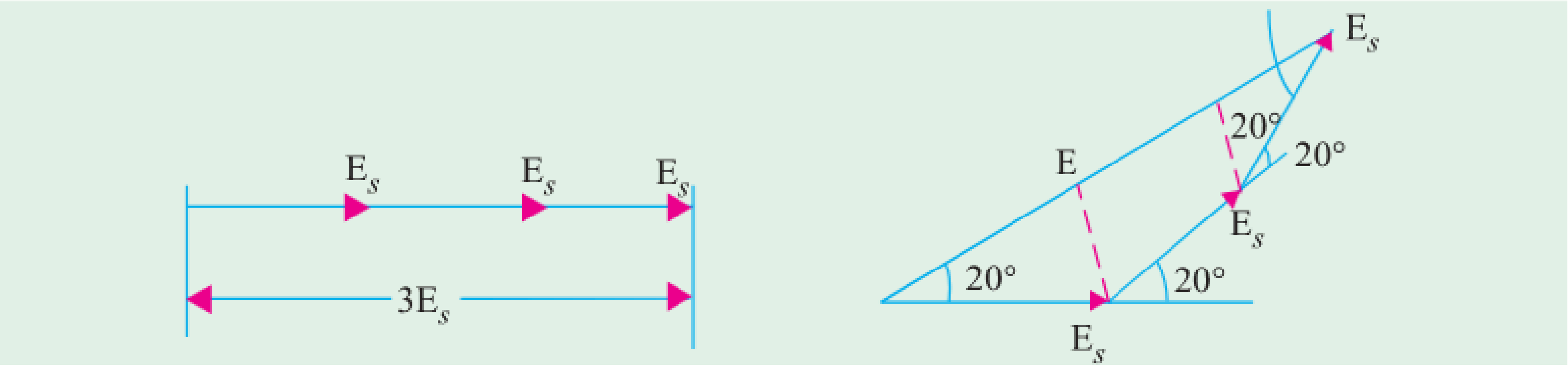
$$
\begin{aligned}
E &= E_S \cos 20° + E_S + E_S \cos 20° \\
&= 2\,E_S \cos 20° + E_S \\
&= 2\,E_S \times 0.9397 + E_S = 2.88\,E_S
\end{aligned}
$$

The distribution factor (k_d) is defined as

$$= \frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$$

In the present case

$$k_d = \frac{\text{e.m.f. with winding in 3 slots/pole/phase}}{\text{e.m.f. with winding in 1 slots/pole/phase}} = \frac{E}{3\,E_S} = \frac{2.88\,E_S}{3\,E_S} = 0.96$$



WEEK 03

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Equation of Induced E.M.F

Let Z = No. of conductors or coil sides in series/phase

= $2T$ — where T is the No. of coils or turns per phase (remember one turn or coil has two sides)

P = No. of poles

f = frequency of induced e.m.f. in Hz

Φ = flux/pole in webers

k_d = distribution factor = $\dfrac{\sin m\beta/2}{m \sin \beta/2}$

k_c or k_p = pitch or coil span factor = $\cos \alpha/2$

k_f = from factor = **1.11** —if e.m.f. is assumed sinusoidal

N = rotor r.p.m.

In one revolution of the rotor (*i.e.* in 60./N second) each stator conductor is cut by a flux of ΦP webers.

$\therefore$ $d\Phi = \Phi P$ and $dt = 60/N$ second

$\therefore$ Average e.m.f. induced per conductor = $\dfrac{d\Phi}{dt} = \dfrac{\Phi P}{60/N} = \dfrac{\Phi NP}{60}$

Now, we know that $f = PN/120$ or $N = 120\,f/P$

Substituting this value of N above, we get

Average e.m.f. per conductor $= \dfrac{\Phi P}{60} \times \dfrac{120\,f}{P} = 2f\,\Phi$ volt

If there are Z conductors in series/phase, then Average e.m.f./phase = $2f\,\Phi Z$ volt = $4f\Phi T$ volt

R.M.S. value of e.m.f./phase $= 1.11 \times 4f\Phi T = 4.44 f\Phi T$ volt*.

Effects of Harmonics on the Pitch and Distribution Factor

(*a*) If the short-pitch angle or chording angle is α degrees (electrical) for the fundamental flux wave, then its values for different harmonics are

for 3rd harmonic $= 3\,\alpha$; for 5th harmonic $= 5\,\alpha$ and so on.

$\therefore$ pitch-factor, $k_c = \cos \alpha/2$ —for fundamental

$= \cos 3\alpha/2$ —for 3rd harmonic

$= \cos 5\alpha/2$ —for 5th harmonic etc.

(*b*) Similarly, the distribution factor is also different for different harmonics. Its value becomes

$$k_d = \frac{\sin n\,m\,\beta/2}{m \sin n\,\beta/2}$$ where n is the order of the harmonic

for fundamental, $n = 1$ $\qquad k_{d1} = \dfrac{\sin m\,\beta/2}{m \sin \beta/2}$

for 3rd harmonic, $n = 3$ $\qquad k_{d3} = \dfrac{\sin 3\,m\,\beta/2}{m \sin 3\,\beta/2}$

for 5th harmonic, $n = 5$ $\qquad k_{d5} = \dfrac{\sin 5\,m\,\beta/2}{m \sin 5\,\beta/2}$

(*c*) Frequency is also changed. If fundamental frequency is 50 Hz *i.e.* $f_1 = 50$ Hz then other frequencies are :

3rd harmonic, $f_3 = 3 \times 50 = 150$ Hz, 5th harmonic, $f_5 = 5 \times 50 = 250$ Hz etc.

Factors Affecting Alternator Size

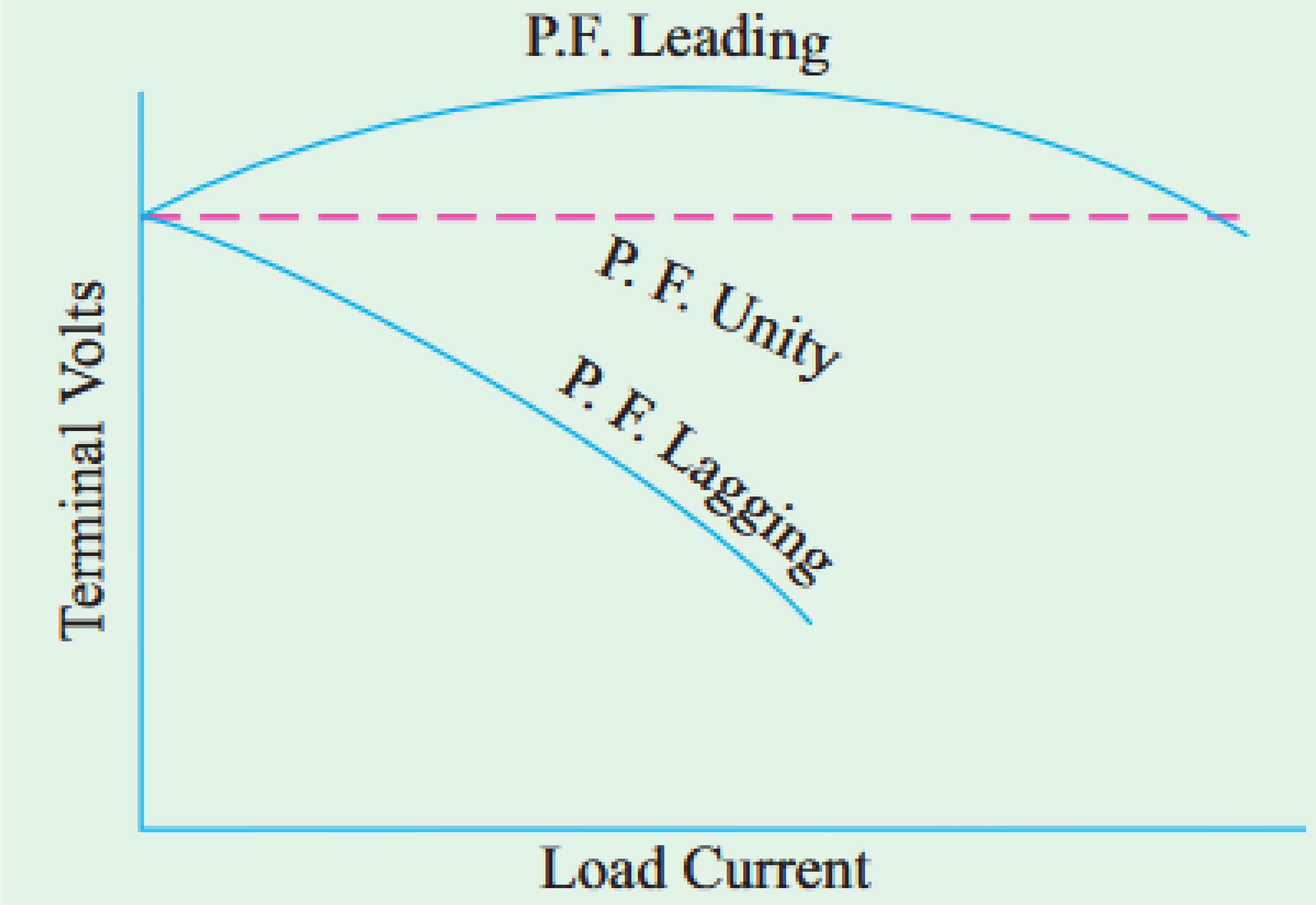
Factor	Effect
Load	Affects voltage regulation and power factor.
Power Factor	Influences armature reaction and voltage stability.
Armature Reaction	Weakens or strengthens main flux depending on load.
Speed	Directly impacts frequency and output voltage.
Field Excitation	Controls terminal voltage and stability.
Temperature	Increases losses and reduces magnetic flux.
Mechanical Factors	Misalignment and vibration cause performance issues.
Losses	Copper, core, and mechanical losses affect overall efficiency.

Voltage Regulation

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends ***not only on the load but also on the load power factor.***

The voltage regulation of an alternator is defined as "the rise in voltage when full-load is removed (field excitation and speed remaining the same) divided by the rated terminal voltage."

$$\therefore \text{ \% regulation 'up'} = \frac{E_0 - V}{V} \times 100$$



WEEK 04

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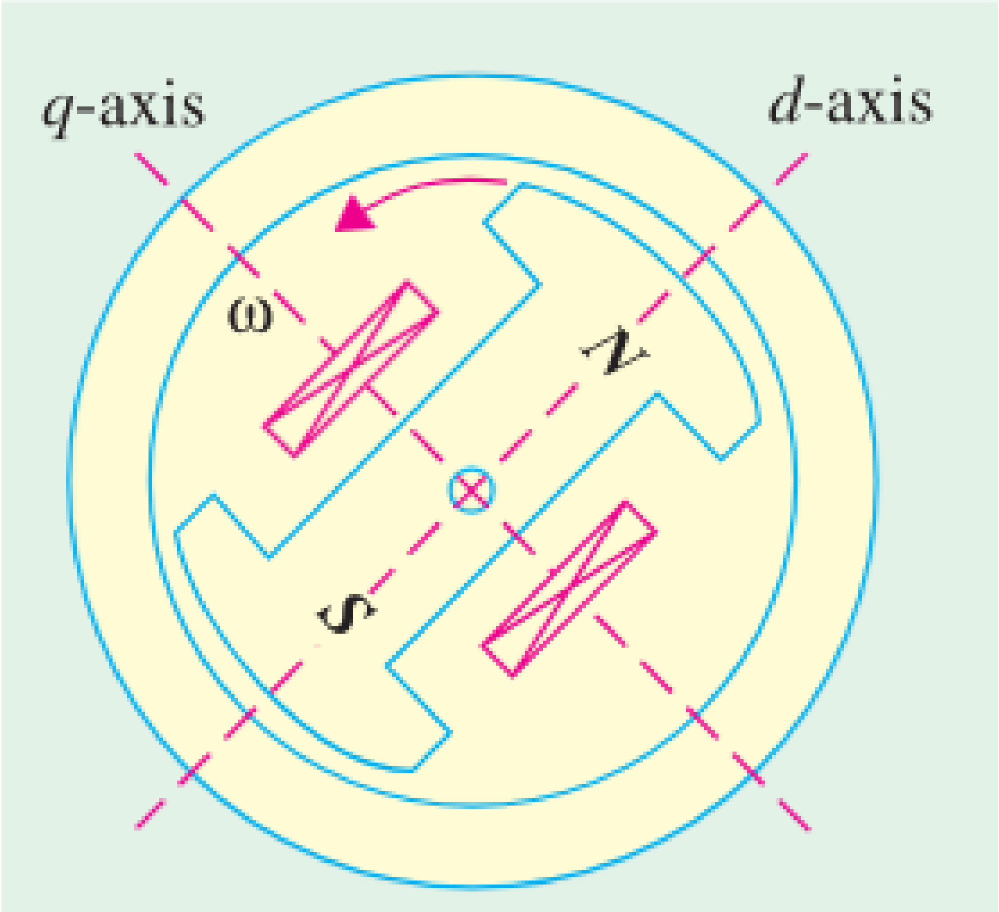
Operation of a Salient Pole Synchronous Machine

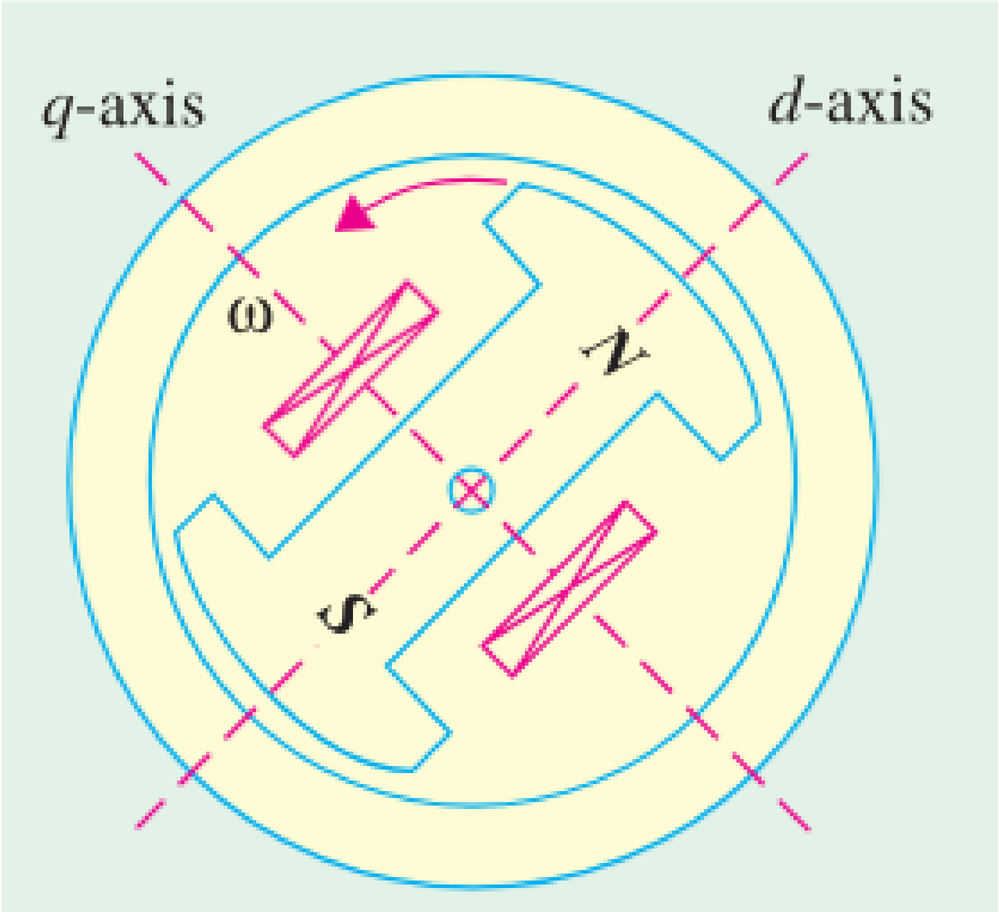
The operation of a salient pole synchronous machine involves the interaction of its rotor and stator magnetic fields to generate or consume electrical power. The machine consists of a salient (protruding) pole rotor, which is typically used in low-speed applications such as hydroelectric power plants. When the machine operates as a generator, the rotor is excited by a DC current, creating a magnetic field. This rotor is driven at synchronous speed by a prime mover (e.g., a turbine), and the rotating magnetic field induces an alternating voltage in the stator windings through electromagnetic induction.



In synchronous operation, the rotor magnetic field rotates at the same speed as the stator's rotating magnetic field. The salient pole structure causes the machine to exhibit anisotropy—its magnetic reluctance is not uniform in all directions. This results in two components of the electromagnetic torque: the synchronous torque, due to interaction between the rotor and stator fields, and the reluctance torque, caused by the alignment tendency of the rotor's poles with the stator's field to minimize magnetic reluctance.

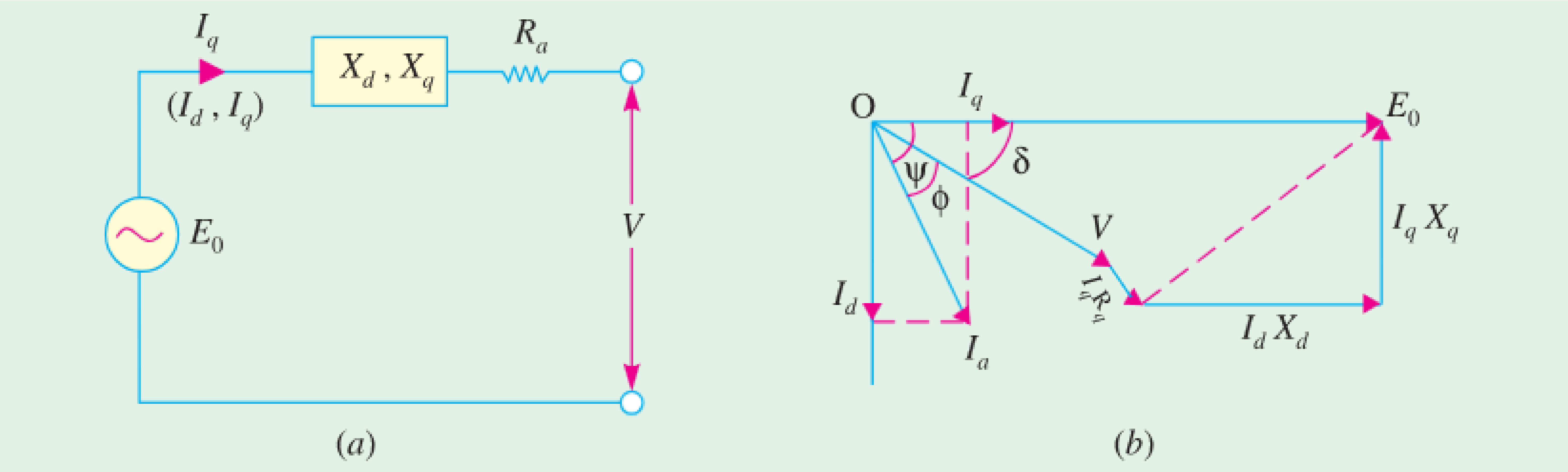
Operation of a Salient Pole Synchronous Machine

When operating as a motor, the salient pole synchronous machine is supplied with three-phase AC power to the stator, producing a rotating magnetic field. The rotor, excited by DC current, aligns itself with the rotating field and runs at synchronous speed. To start, an auxiliary method such as a damper winding or external prime mover is used because synchronous motors cannot self-start.



The salient pole design offers advantages such as better torque characteristics at low speeds and high efficiency in applications where the machine speed is relatively slow. However, it exhibits a non-uniform air gap, leading to a more complex analysis of its performance, including considerations of direct-axis and quadrature-axis reactances (d-axis and q-axis). These reactances impact the machine's voltage regulation and stability, making it essential to account for them in operational analysis.

Phasor Diagram for a Salient Pole Synchronous Machine

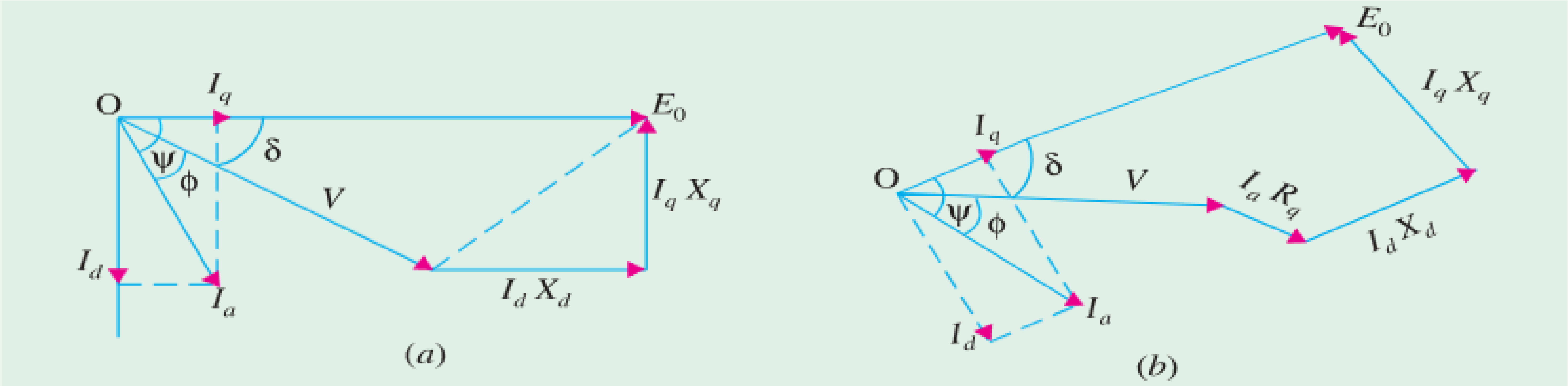


Calculations from Phasor Diagram

In Fig. dotted line AC has been drawn perpendicular to I_a and CB is perpendicular to the phasor for E_0. The angle $ACB = \psi$ because angle between two lines is the same as between their perpendiculars. It is also seen that

$$I_d = I_a \sin\psi \,;\, I_q = I_a \cos\psi \,;\, \text{hence, } I_a = I_q / \cos\psi$$

In ΔABC, $\quad BC/AC = \cos\psi$ or $AC = BC/\cos\psi = I_q X_q / \cos\psi = I_a X_q$



Calculations from Phasor Diagram

From $\Delta\, ODC$, we get

$$\tan\psi = \frac{AD + AC}{OE + ED} = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a} \qquad \text{—generating}$$

$$= \frac{V\sin\phi - I_a X_q}{V\sin\phi - I_a R_a} \qquad \text{—motoring}$$

The angle ψ can be found from the above equation. Then, $\delta = \psi - \phi$ (generating) and $\delta = \phi - \psi$ (motoring)

As seen from Fig. 37.73, the excitation voltage is given by

$E_0 = V\cos\delta + I_q R_a + I_d X_d$ —generating

Calculations from Phasor Diagram

$= V\cos\delta - I_q R_a - I_d X_d$ —motoring

Note. Since angle ϕ is taken positive for lagging p.f., it will be taken negative for leading p.f.

If we neglect the armatrue resistance as shown in Fig. then angle δ can be found directly as under :

$\psi = \phi + \delta$ (generating)

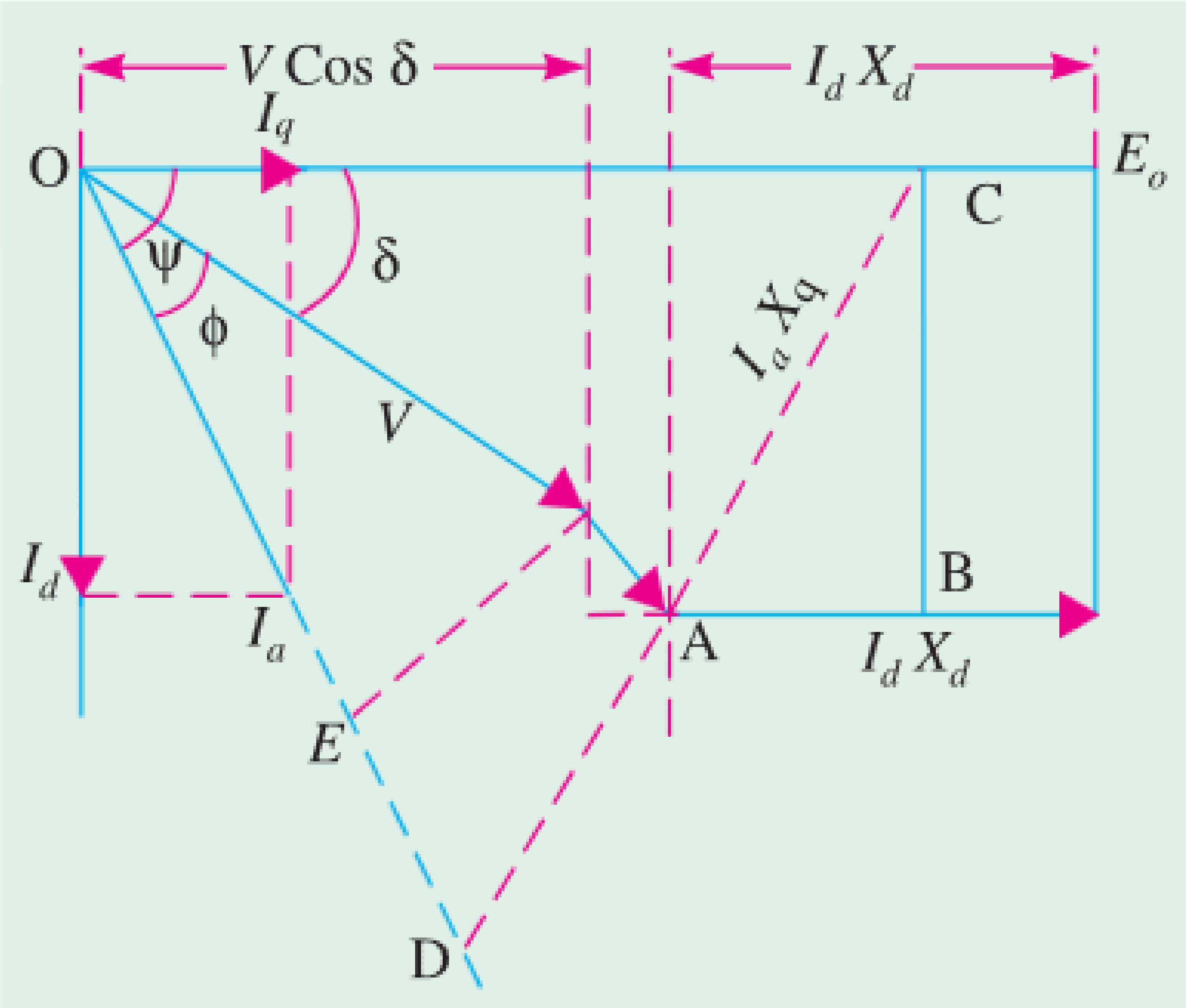
and $\psi = \phi - \delta$ (motoring).

In general, $\psi = (\phi \pm \delta)$.

$I_d = I_a \sin\psi$

$= I_a \sin(\phi \pm \delta)$; $I_q = I_a \cos\psi = I_a \cos(\phi \pm \delta)$

As seen from Fig. 37.73, $V\sin\delta = I_q X_q = I_a X_q \cos(\phi \pm \delta)$



Calculations from Phasor Diagram

$$\therefore \quad V \sin \delta = I_a X_q (\cos \phi \cos \delta \pm \sin \phi \sin \delta)$$

$$\text{or} \quad V = I_a X_q \cos \phi \cot \delta \pm I_a X_q \sin \phi$$

$$\therefore \quad I_a X_q \cos \phi \cot \delta = V \pm I_a X_q \sin \phi$$

$$\therefore \quad \tan \delta = \frac{I_a X_q \cos \phi}{V \pm I_a X_q \sin \phi}$$

In the above expression, plus sign is for synchronous generators and minus sign for synchronous motors.

Similarly, when R_a is neglected, then,

$$E_0 = V \cos \delta \pm I_d X_d$$

However, if R_a and hence $I_a R_a$ drop is not negligible then,

$$E_0 = V \cos \delta + I_q R_a + I_d X_d \quad \text{—generating}$$

$$= V \cos \delta - I_q R_a - I_d X_d \quad \text{—motoring}$$

WEEK 05

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Power Developed by a Synchronous Generator

If we neglect R_a and hence Cu loss, then the power developed (P_d) by an alternator is equal to the power output (P_{out}). Hence, the per phase power output of an alternator is

$$P_{out} = VI_a \cos\phi = \text{power developed } (p_d) \qquad \text{...}(i)$$

Now, as seen from Fig., 37.72 (*a*), $I_q X_q = V \sin\delta$; $I_d X_d = E_0 - V\cos\delta$ ...(*ii*)

Also,
$$I_d = I_a \sin(\phi+\delta);\ I_q = I_a \cos(\phi+\delta) \qquad \text{...}(iii)$$

Substituting Eqn. (*iii*) in Eqn. (*ii*) and solving for $I_a \cos\phi$, we get

$$I_a \cos\phi = \frac{V}{X_d}\sin\delta + \frac{V}{2X_q}\sin 2\delta - \frac{V}{2X_d}\sin 2\delta$$

Finally, substituting the above in Eqn. (*i*), we get

$$P_d = \frac{E_0 V}{X_d}\sin\delta + \frac{1}{2}V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta = \frac{E_0 V}{X_d}\sin\delta + \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\delta$$

The total power developed would be three times the above power.

Parallel Operation of Alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as ***synchronizing***. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of constant voltage and constant frequency. Often the electrical system to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to ***infinite*** bus-bars.

It is never advisable to connect a stationary alternator to live bus-bars, because, stator induced e.m.f. being zero, a short-circuit will result. For proper synchronization of alternators, the following three conditions must be satisfied :

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage.

2. The speed of the incoming machine must be such that its frequency ($= PN/120$) equals bus-bar frequency.

3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage. It means that the switch must be closed at (or very near) the instant the two voltages have correct phase relationship.

WEEK 06

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Synchronizing of Alternators

Synchronizing of Alternators refers to the process of connecting an alternator to a live electrical grid or another alternator already in operation. Synchronization ensures that the incoming alternator operates smoothly in parallel without causing disturbances to the system. This process is crucial for maintaining system stability, sharing loads, and ensuring efficient operation.

Conditions for Synchronizing Alternators

Voltage Equality:
The terminal voltage of the incoming alternator must match the voltage of the existing system or running alternator.
Frequency Equality:
The frequency of the incoming alternator must be the same as that of the grid or running alternator.
Phase Sequence:
The phase sequence (order of phases: R-Y-B) of the incoming alternator must match the phase sequence of the existing system.
Phase Angle Matching:
The phase angle of the voltage waveform of the incoming alternator must align with the voltage waveform of the system.

These conditions ensure that there is minimal voltage difference between the alternator and the grid to avoid circulating currents and mechanical stresses.

Synchronizing of Alternators

Methods of Synchronizing Alternators

Three Lamp Method:
Three lamps are connected across corresponding phases of the alternator and the grid.
When the lamps darken and brighten simultaneously, the phase sequence is correct, and synchronization occurs when all lamps are dark (zero potential difference).

Synchroscope Method:
A synchroscope is used to display the difference in frequency and phase between the incoming alternator and the grid.
The pointer on the synchroscope rotates; the alternator is synchronized when the pointer is stationary and pointing to the "12 o’clock" position.

Automatic Synchronization:
Modern systems use automatic synchronizers and sensors to adjust the voltage, frequency, and phase automatically.

Synchronizing of Alternators

Procedure for Synchronizing Alternators
Start the Alternator and bring it to rated speed using the prime mover.
Match the voltage of the incoming alternator to the grid voltage using field excitation.
Check and adjust the frequency of the alternator to match the grid frequency.
Verify the phase sequence using the three-lamp method or a synchroscope.
Wait until the phase angle difference is minimal (synchroscope pointer is stationary).
Close the circuit breaker to connect the alternator to the grid.

Importance of Synchronizing Alternators
Ensures smooth and stable parallel operation.
Prevents circulating currents that can damage the alternator windings.
Maintains voltage stability and frequency in the power system.
Enables load sharing among alternators.

Synchronizing Current

Synchronizing Current refers to the current that flows when an alternator is connected to a power system or another alternator without perfect synchronization. This current arises due to differences in voltage magnitude, phase angle, or frequency between the incoming alternator and the running system. If these differences are significant, the synchronizing current can be very large, potentially causing severe mechanical and electrical stress.

Synchronizing Power

When an alternator is operating in parallel with a power system, its rotor aligns with the rotating magnetic field of the stator, maintaining synchronism. If the rotor experiences a small displacement or phase shift, a torque is developed due to the difference in phase angle between the alternator's internal voltage and the system voltage. This torque produces synchronizing power, which works to restore the rotor back to its original position.

Synchronizing power is directly proportional to the change in phase angle (δ\deltaδ) and helps maintain the stability of the alternator in the power system.

Alternators Connected to Infinite Bus-bars

Now, consider the case of an alternator which is connected to infinite bus-bars. The expression for P_{SY} given above is still applicable but with one important difference *i.e.* impedance (or reactance) of only that one alternator is considered (and not of two as done above). Hence, expression for synchronizing power in this case becomes

$$E_r = \alpha E \qquad \text{—as before}$$

$$I_{SY} = E_r/Z_S \cong E_r/X_S = \alpha E/X_S \qquad \text{—if } R_a \text{ is negligible}$$

$\therefore$ Synchronizing power $$P_{SY} = E\, I_{SY} = E.\alpha E/Z_S = \alpha E^2/Z_S \cong \alpha E^2/X_S \qquad \text{— per phase}$$

Now, $$E/Z_S \cong E/X_S = \text{S.C. current } I_{SC}$$

$\therefore$ $$P_{SY} = \alpha E^2/XS = \alpha E.E/X_S = \alpha E.I_{SY} \qquad \text{—per phase}$$

(more accurately, $P_{SY} = \alpha E^2 \sin\theta/X_S = \alpha E.I_{SC}.\sin\theta$)

Total synchronizing power for three phases $= 3\, P_{SY}$

Synchronizing Torque

Let T_{SY} be the synchronizing torque per phase in newton-metre (N-m)

(*a*) When there are two alternators in parallel

$$\therefore \qquad T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \quad \therefore \; T_{SY} = \frac{P_{SY}}{2\pi N_S / 60} = \frac{\alpha E^2 / 2X_S}{2\pi N_S / 60} \text{ N-m}$$

Total torque due to three phases. $= \dfrac{3P_{SY}}{2\pi N_S / 60} = \dfrac{3\alpha E^2 / 2X_S}{2\pi N_S / 60}$ N-m

(*b*) Alternator connected to infinite bus-bars

$$T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \quad \text{or} \quad T_{SY} = \frac{P_{SY}}{2\pi N_S / 60} = \frac{\alpha E^2 / X_S}{2\pi N_S / 60} \text{ N-m}$$

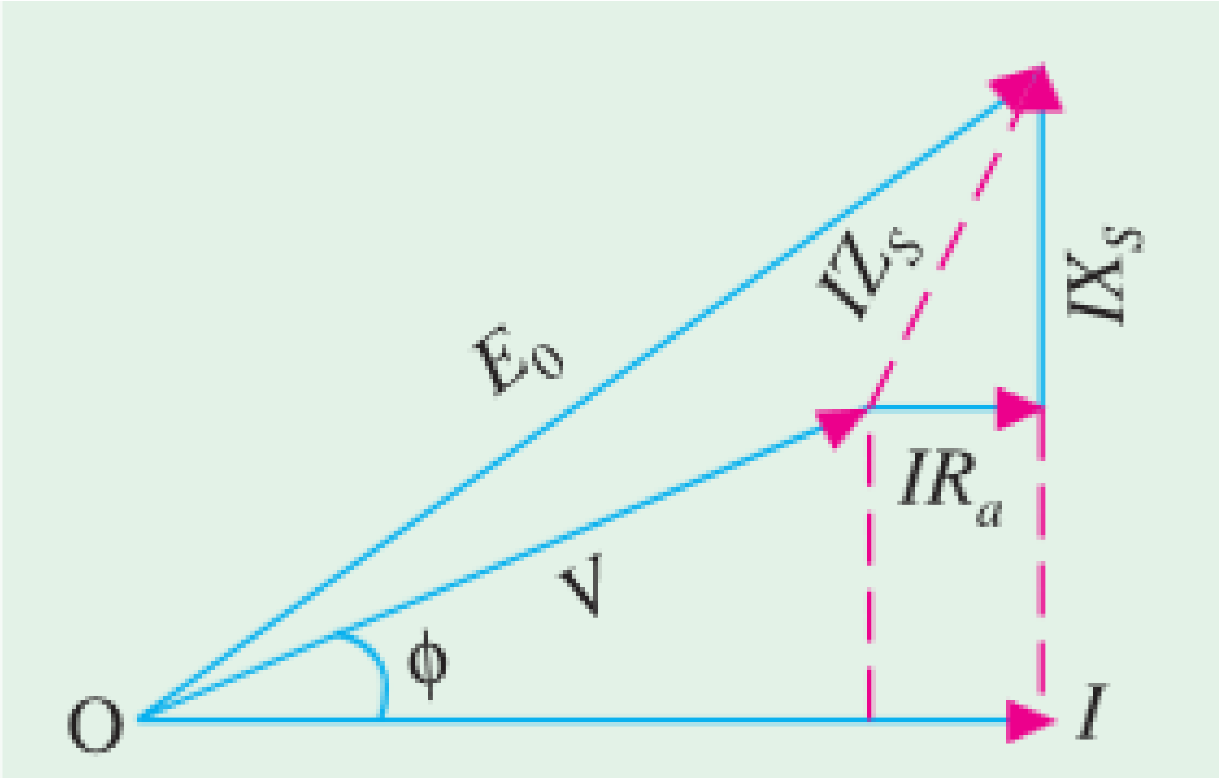
Again, torque due to 3 phase $= \dfrac{3P_{SY}}{2\pi N_S / 60} = \dfrac{3\alpha E^2 / X_S}{2\pi N_S / 60}$ N-m

where N_S = synchronous speed in r.p.m. $= 120\, f/P$

Effect of Load on Synchronizing Power

In this case, instead of $P_{SY} = \alpha\, E^2/X_S$, the approximate value of synchronizing power would be $\cong \alpha EV/X_S$ where V is bus-bar voltage and E is the alternator induced e.m.f. per phase. The value of $E = V + IZ_S$

$$E = (V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$$



WEEK 07

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Parallel Operation of Two Alternators

Consider two alternators with identical speed/load characteristics connected in parallel as shown in Fig. The common terminal voltage **V** is given by

$$\mathbf{V} = \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 = \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2$$

$$\therefore \quad \mathbf{E}_1 - \mathbf{E}_2 = \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2$$

Also $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$ and $\mathbf{V} = \mathbf{IZ}$

$$\therefore \quad \mathbf{E}_1 = \mathbf{I}_1\mathbf{Z}_1 + \mathbf{IZ} = \mathbf{I}_1(\mathbf{Z}+\mathbf{Z}_1) + \mathbf{I}_2\mathbf{Z}$$

$$\mathbf{E}_2 = \mathbf{I}_2\mathbf{Z}_2 + \mathbf{IZ} = \mathbf{I}_2(\mathbf{Z}+\mathbf{Z}_2) + \mathbf{I}_1\mathbf{Z}$$

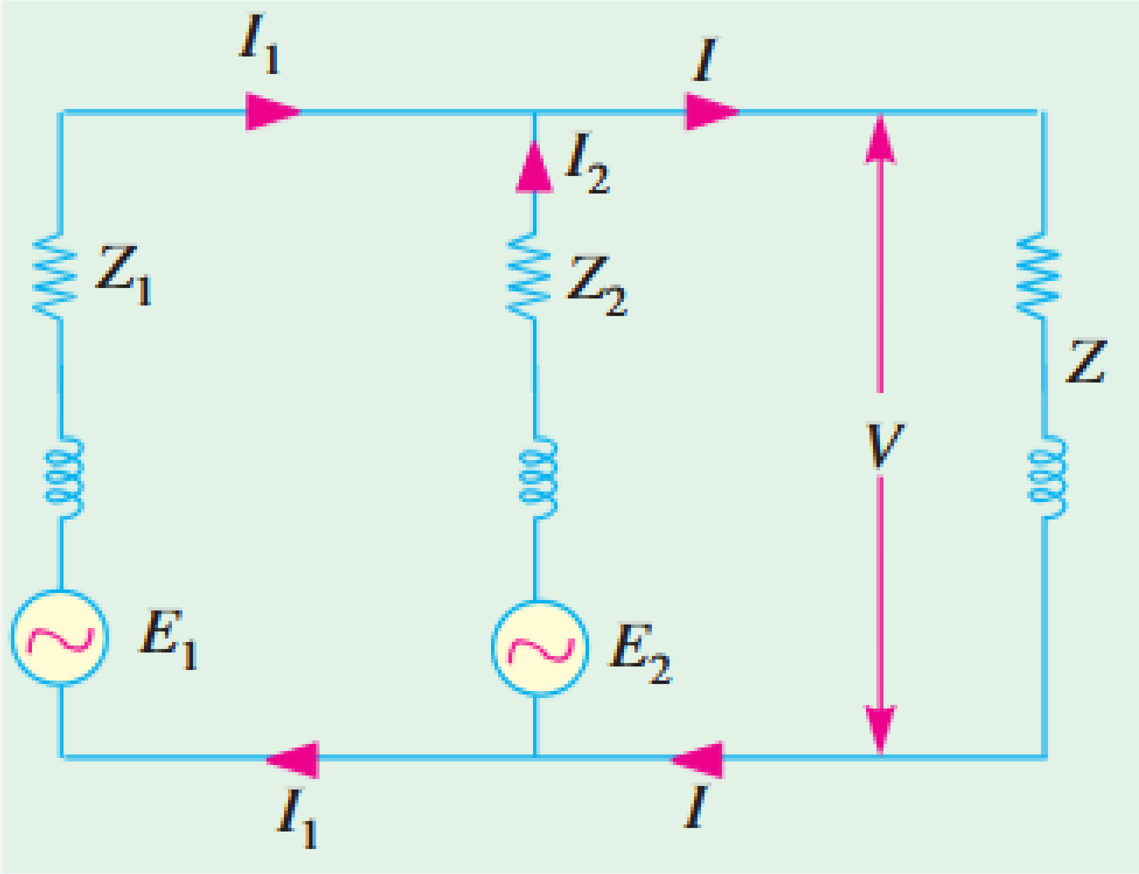
$$\therefore \quad \mathbf{I}_1 = \frac{(\mathbf{E}_1 - \mathbf{E}_2)\,\mathbf{Z} + \mathbf{E}_1\mathbf{Z}_2}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{I}_2 = \frac{(\mathbf{E}_2 - \mathbf{E}_1)\,\mathbf{Z} + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2};$$

$$\mathbf{I} = \frac{\mathbf{E}_1\mathbf{Z}_2 + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{V} = \mathbf{IZ} = \frac{\mathbf{E}_1\,\mathbf{Z}_2 + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2 + (\mathbf{Z}_1\mathbf{Z}_2/\mathbf{Z})}\,;\mathbf{I}_1 = \frac{\mathbf{E}_1 - \mathbf{V}}{\mathbf{Z}_1}\,;\mathbf{I}_2 = \frac{\mathbf{E}_2 - \mathbf{V}}{\mathbf{Z}_2}$$

The circulating current under no-load condition is $\mathbf{I}_C = (\mathbf{E}_1 - \mathbf{E}_2)/(\mathbf{Z}_1 + \mathbf{Z}_2)$.



Parallel Operation of Two Alternators

Using Admittances

The terminal Voltage may also be expressed in terms of admittances as shown below:

$$V = IZ = (I_1 + I_2)Z \qquad \therefore\ I_1 + I_2 = V/Z = VY \qquad ..(i)$$

Also $\quad I_1 = (E_1 - V)/Z_1 = (E_1 - V)Y_1\,; \qquad I_2 = (E_2 - V)/Z_2 = (E_2 - V)Y_2$

$$\therefore \quad I_1 + I_2 = (E_1 - V)\,Y_1 + (E_2 - V)Y_2 \qquad ...(ii)$$

From Eq. (*i*) and (*ii*), we get

$$VY = (E_1 - V)Y_1 + (E_2 - V)Y_2 \quad \text{or} \quad V = \frac{E_1\,Y_1 + E_2 Y_2}{Y_1 + Y_2 + Y}$$

Using Parallel Generator Theorem

$$V = IZ = (I_1 + I_2)\,Z = \left(\frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2}\right)Z$$

$$= \left(\frac{E_1}{Z_1} + \frac{E_2}{Z_2}\right)Z - V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)Z$$

$$\therefore \quad V\left(\frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2}\right) = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} = I_{SC1} + I_{SC2} = I_{SC}$$

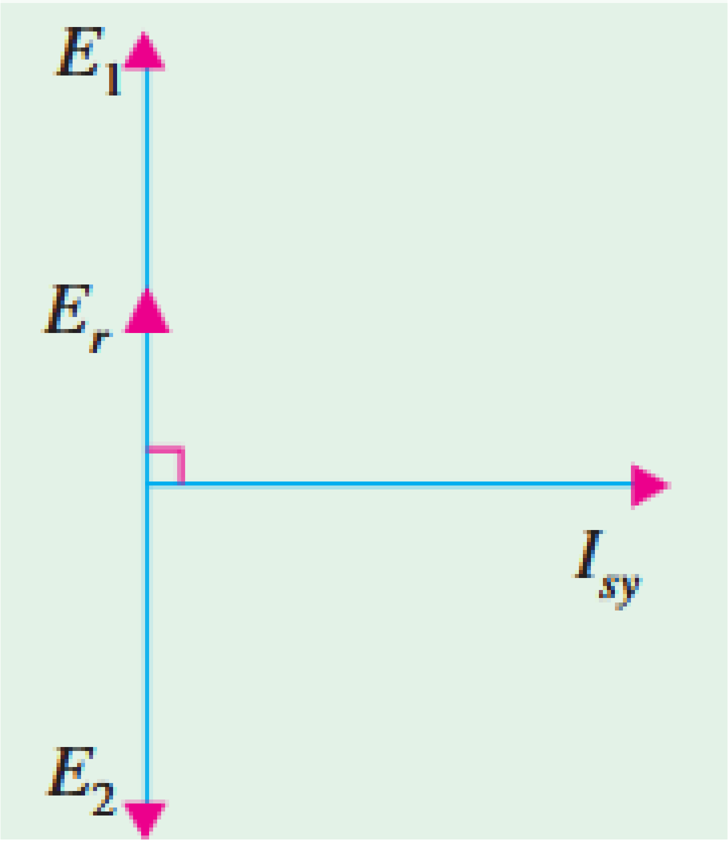
where I_{SC1} and I_{SC2} are the short-circuit currents of the two alternators.

If $\quad \dfrac{1}{Z_0} = \left(\dfrac{1}{Z} + \dfrac{1}{Z_1} + \dfrac{1}{Z_2}\right)$; then $V \times \dfrac{1}{Z_0} = I_{SC}$ or $V = Z_0\, I_{SC}$

Effect of Unequal Voltage

Let us consider two alternators, which are running exactly in-phase (relative to the external circuit) but which have slightly unequal voltages, as shown in Fig.

If E_1 is greater than E_2, then their resultant is $E_r = (E_1 - E_2)$ and is in-phase with E_1. This E_r or E_{SY} set up a local synchronizing current I_{SY} which (as discussed earlier) is almost 90° behind E_{SY} and hence behind E_1 also. This lagging current produces demagnetising effect on the first machine, hence E_1 is reduced. The other machine runs as a synchronous motor, taking almost 90° leading current. Hence, its field is strengthened due to magnetising effect of armature reaction (Art. 37.16). This tends to increase E_2. These two effects act together and hence lessen the inequalities between the two voltages and tend to establish stable conditions.



Maximum Power Output

The power output per phase is

$$P = VI \cos\phi = \frac{VIX_S \cos\phi}{X_S}$$

Now, from $\Delta\, OBC$, we get

$$\frac{IX_S}{\sin\alpha} = \frac{E}{\sin(90+\phi)} = \frac{E}{\cos\phi}$$

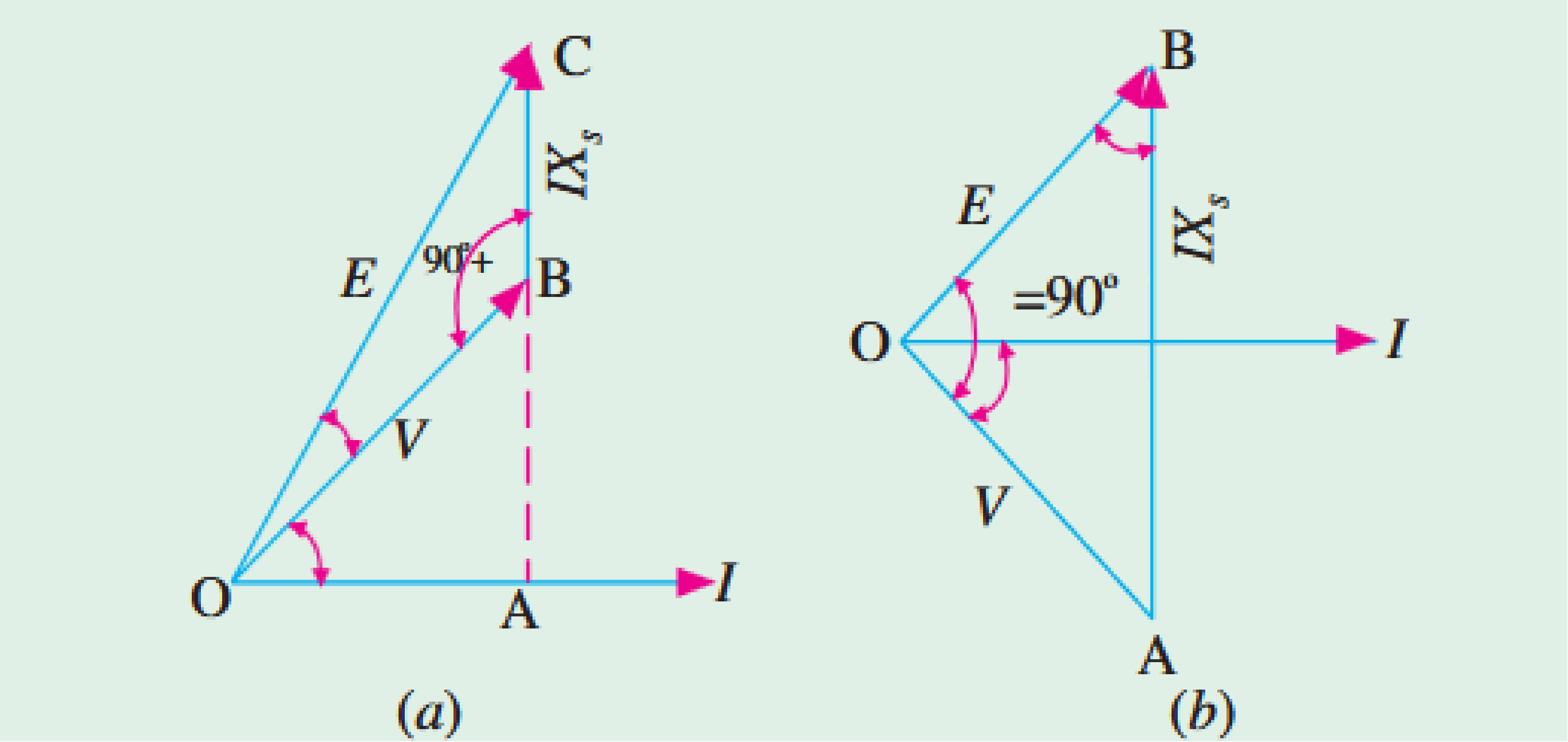
$$IX_S \cos\phi = E \sin\alpha$$

$$\therefore \qquad P = \frac{EV \sin\alpha}{X_S}$$

Power becomes maximum when $\alpha = 90°$, if V, E and X_S are regarded as constant (of course, E is fixed by excitation).

$$\therefore \qquad P_{max} = EV/X_S$$

It will be seen from Fig. (*b*) that under maximum power output conditions, I leads V by ϕ and since IX_S leads I by 90°, angle ϕ and hence $\cos\phi$ is fixed $= E/IX_S$



Maximum Power Output

Now, from right-angled ΔAOB, we have that $IX_S = \sqrt{E^2 + V^2}$. Hence, p.f. corresponding to maximum power output is

$$\cos\phi = \frac{E}{\sqrt{E^2 + V^2}}$$

The maximum power output per phase may also be written as

$$P_{max} = VI_{max}\cos\phi = VI_{max}\frac{E}{\sqrt{E^2 + V^2}}$$

where I_{max} represents the current/phase for maximum power output.

If I_f is the full-load current and $\% X_S$ is the percentage synchronous reactance, then

$$\% X_S = \frac{I_f X_S}{V} \times 100 \qquad \therefore \quad \frac{V}{X_S} = \frac{I_f \times 100}{\% X_S}$$

Now, $$P_{max} = VI_{max}\frac{E}{\sqrt{E^2 + V^2}} = \frac{EV}{X_S} = \frac{EI_f \times 100}{\% X_S}$$

Two things are obvious from the above equations.

(*i*) $$I_{max} = \frac{100\, I_f}{\% X_S} \times \frac{\sqrt{E^2 + V^2}}{V}$$

Substituting the value of $\% X_S$ from above,

$$I_{max} = \frac{100\, I_f}{100\, I_f X_S} \times V \times \frac{\sqrt{E^2 + V^2}}{V} = \frac{\sqrt{E^2 + V^2}}{X_S}$$

Maximum Power Output

(*ii*) $$P_{max} = \frac{100\, EI_f}{\%\, X_S} = \frac{E}{V} \cdot \frac{100}{\%\, X_S} \times VI_f \text{ per phase}$$

$$= \frac{E}{V} \cdot \frac{100}{\%\, X_S} \times \text{ F.L. power output at u.p.f.}$$

Total maximum power output of the alternator is

$$= \frac{E}{V} \cdot \frac{100}{\%\, X_S} \times \text{ F.L. power output at u.p.f.}$$

WEEK 08

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Mathematical Problems on Alternator

Mathematical problems related to transformers will be practiced and solved during classroom sessions. Problems from the prescribed reference book will be addressed, and additional practice materials will be provided to enhance understanding and proficiency.

WEEK 09

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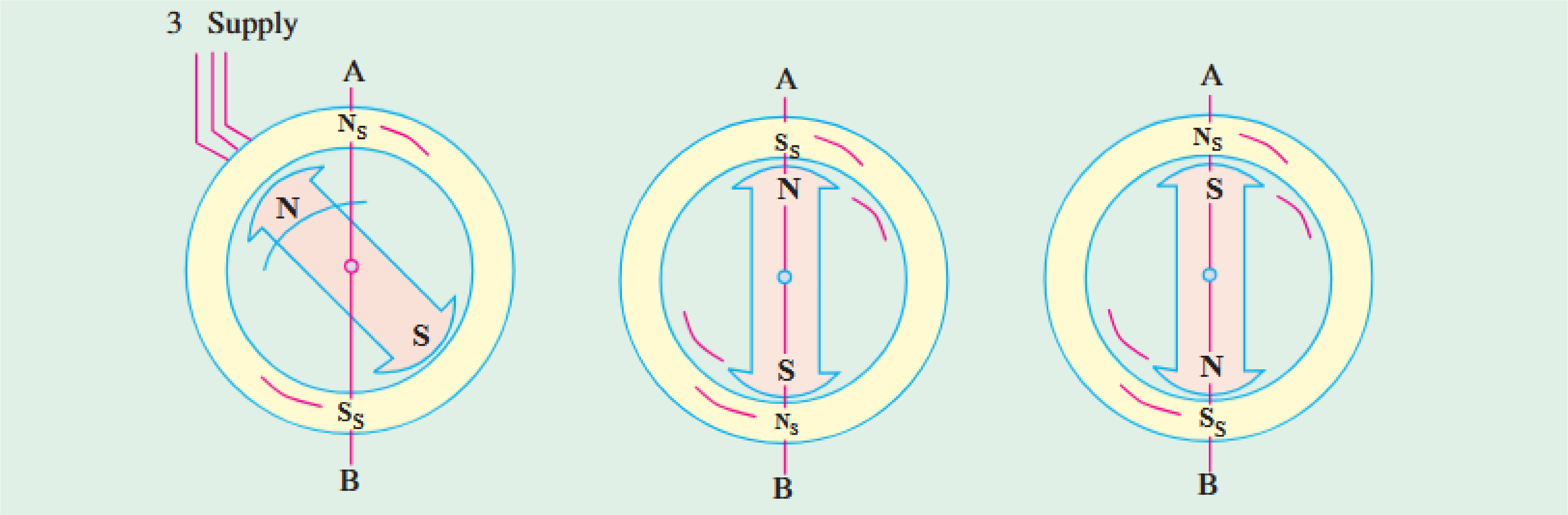
Synchronous Motor

1. It runs either at synchronous speed or not at all *i.e.* while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency (because $Ns = 120 f / P$).
2. It is not inherently self-starting. It has to be run upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.
3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

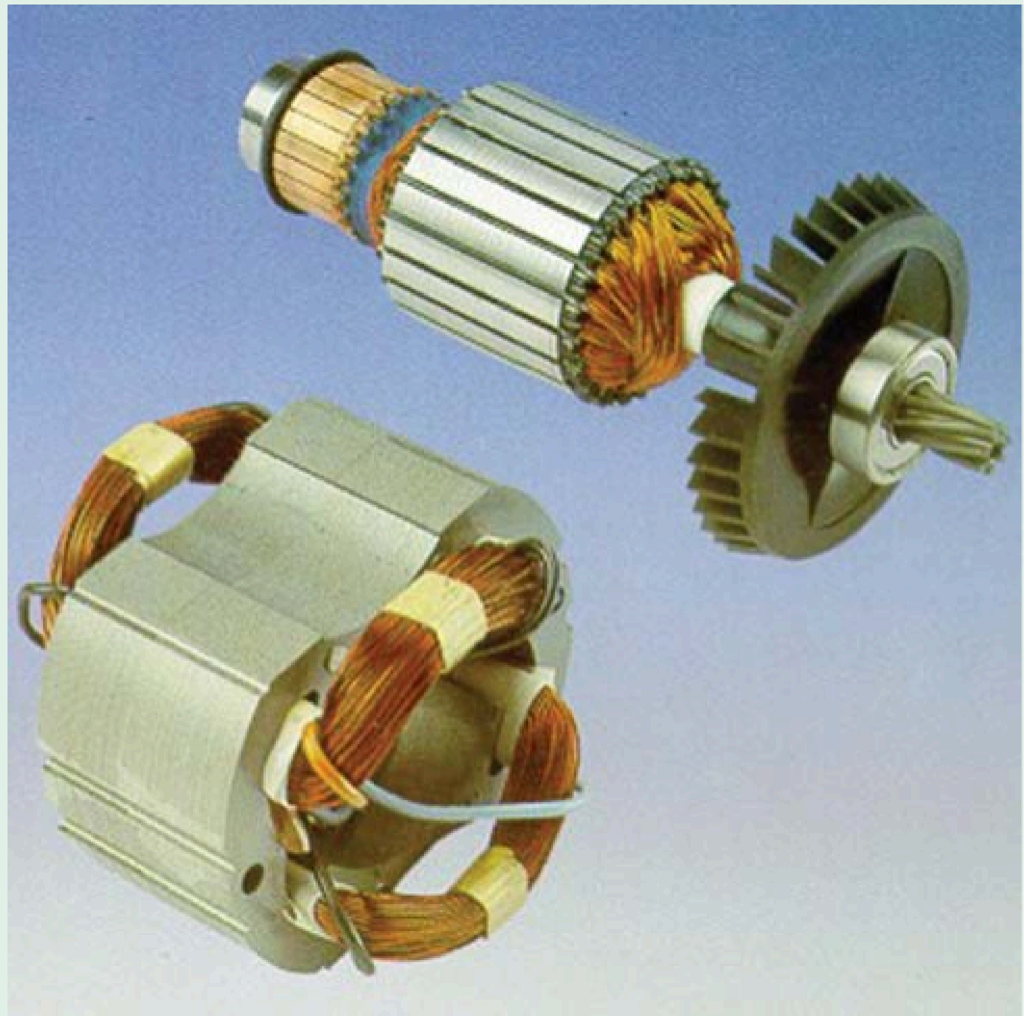
Synchronous motor

Principle of Operation



(a) (b)

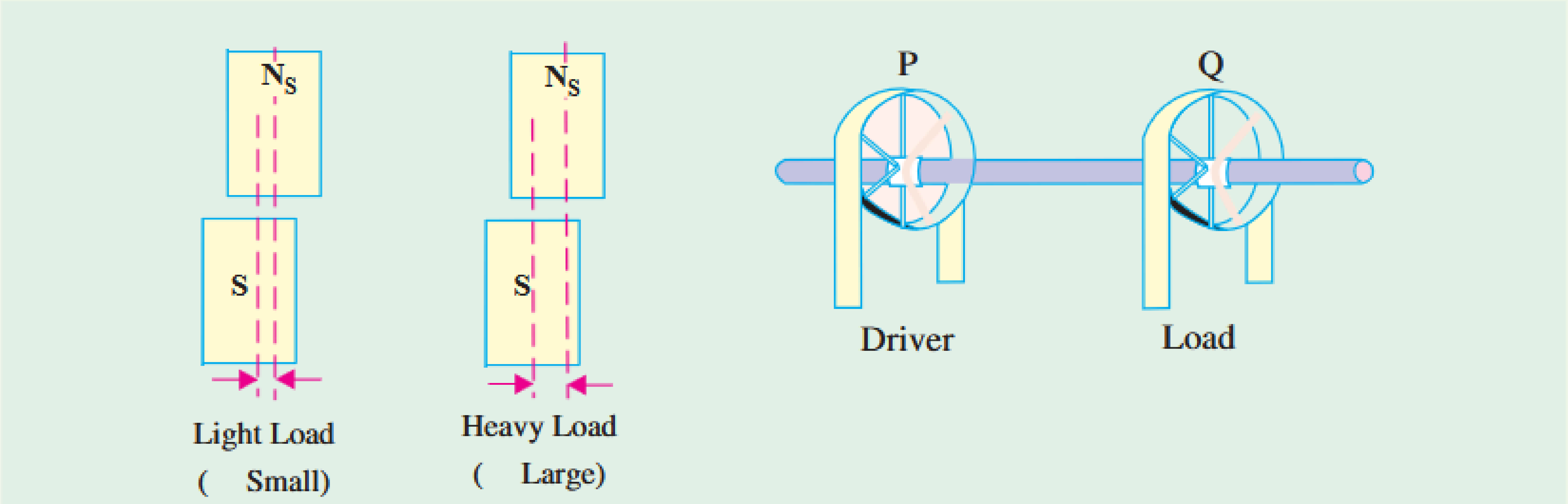
Principle of Operation


The rotor and the stator parts of motor.

The rotor (which is as yet unexcited) is speeded up to synchronous / near synchronous speed by some arrangement and then excited by the d.c. source. The moment this (near) synchronously rotating rotor is excited, it is magnetically locked into position with the stator *i.e.*, the rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all. The synchronous speed is given by the usual relation $N_S = 120 f / P$.

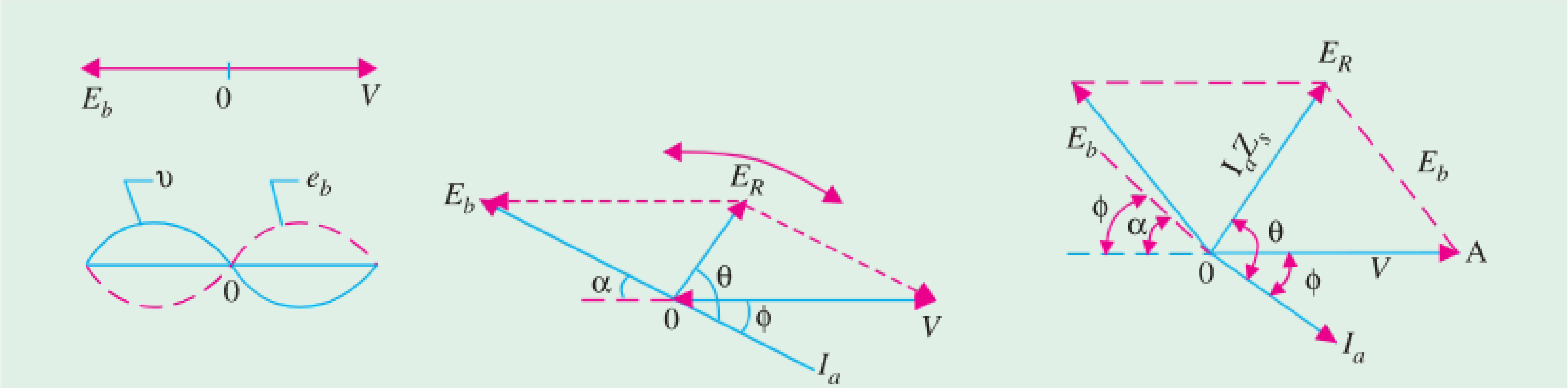
Principle of Operation

However, it is important to understand that the arrangement between the stator and rotor poles is ***not an absolutely rigid one***. As the load on the motor is increased, the rotor progressively tends to fall back ***in phase*** (but *not* in speed as in d.c. motors) by some angle (Fig.) ***but it still continues to run synchronously***. The value of this load angle or coupling angle (as it is called) depends on the amount of load to be met by the motor. In other words, the torque developed by the motor depends on this angle, say, α.



Motor on Load with Constant Excitation

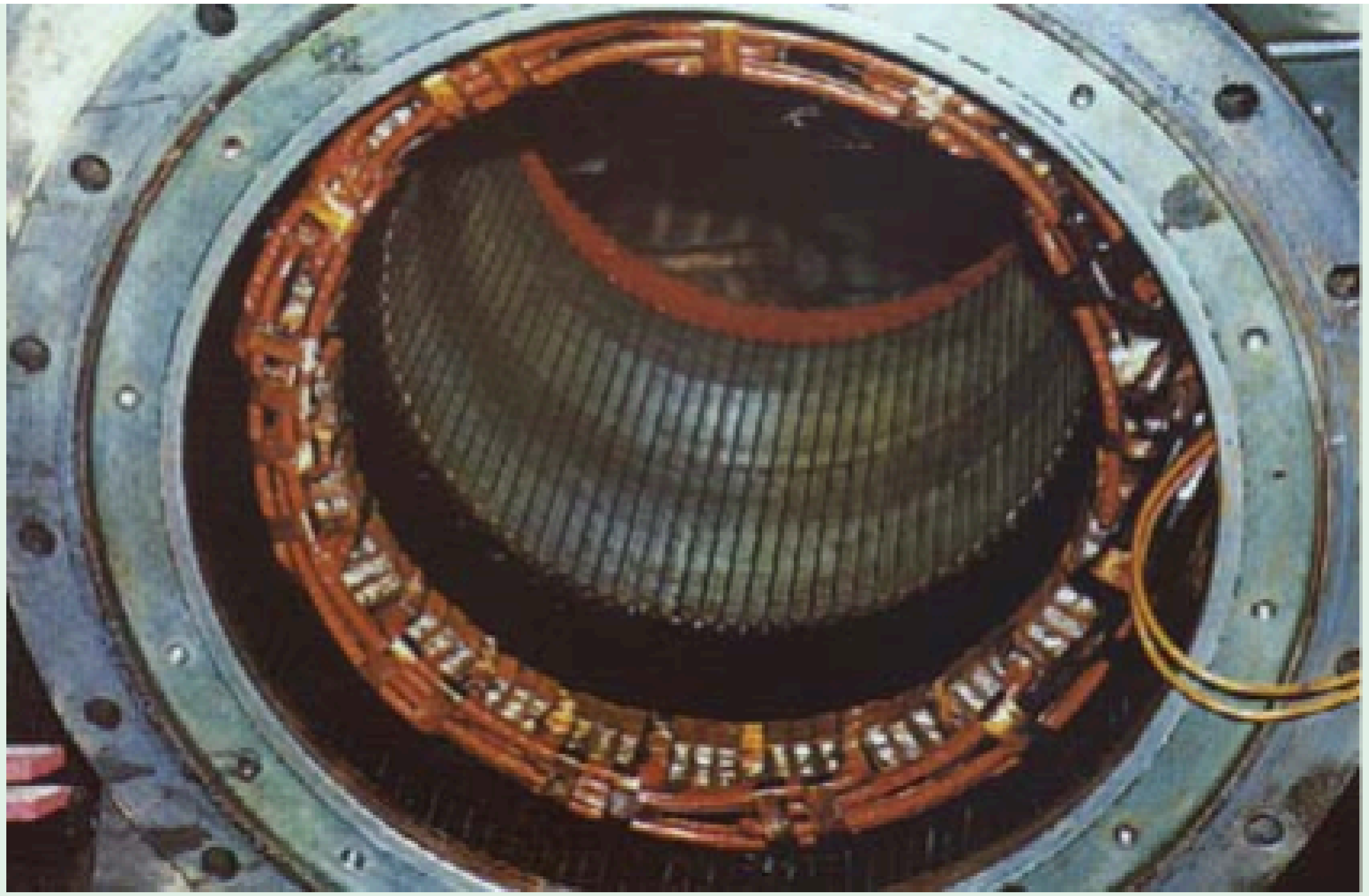
Before considering as to what goes on inside a synchronous motor, it is worthwhile to refer briefly to the d.c. motors. We have seen that when a d.c. motor is running on a supply of, say, V volts then, on rotating, a back e.m.f. E_b is set up in its armature conductors. The resultant voltage across the armature is $(V - E_b)$ and it causes an armature current $I_a = (V - E_b)/R_a$ to flow where R_a is armature circuit resistance. The value of E_b depends, among other factors, on the speed of the rotating armature. The mechanical power developed in armature depends on $E_b I_a$ (E_b and I_a being in opposition to each other).



Motor on Load with Constant Excitation

Fig. shows the condition when the motor (properly synchronized to the supply) is running on ***no-load*** and has ***no losses***.* and is having field excitation which makes $E_b = V$. It is seen that vector difference of E_b and V is zero and so is the armature current. Motor intake is zero, as there is neither load nor losses to be met by it. In other words, the motor just floats.

If motor is on no-load, but it has losses, then the vector for E_b falls back (vectors are rotating anti-clockwise) by a certain small angle α (Fig.), so that a resultant voltage E_R and hence current I_a is brought into existence, which supplies losses.**



Stator of synchronous motor

Power Flow within a Synchronous Motor

Let R_a = armature resistance / phase ; X_S = synchronous reactance / phase

then $\mathbf{Z}_S = \mathbf{R}_a + jX_S$; $\mathbf{I}_a = \dfrac{\mathbf{E}_R}{\mathbf{Z}_S} = \dfrac{\mathbf{V} - \mathbf{E}_b}{\mathbf{Z}_S}$; Obviously, $\mathbf{V} = \mathbf{E}_b + \mathbf{I}_a \mathbf{Z}_S$

The angle θ (known as internal angle) by which I_a lags behind E_R is given by $\tan\theta = X_S / R_a$.

If R_a is negligible, then $\theta = 90°$. Motor input $= V\,I_a \cos\phi$ —per phase

Here, V is applied voltage / phase.

Total input for a star-connected, 3-phase machine is, $P = \sqrt{3}\,V_L \,.\, I_L \cos\phi$.

The mechanical power developed in the rotor is

P_m = back e.m.f. × armature current × cosine of the angle between the two *i.e.*, angle between I_a and E_b reversed.

$= E_b I_a \cos(\alpha - \phi)$ per phase ...Fig. 38.8

Power Flow within a Synchronous Motor

Out of this power developed, some would go to meet iron and friction and excitation losses. Hence, the power available at the shaft would be less than the developed power by this amount.

Out of the input power / phase $V\,I_a \cos\phi$, and amount $I_a^2\,R_a$ is wasted in armature***, the rest $(V.\,I_a \cos\phi - I_a^2\,R_a)$ appears as mechanical power in rotor; out of it, iron, friction and excitation losses are met and the rest is available at the shaft. If power input / phase of the motor is P, then

$$P = P_m + I_a^2\,R_a$$

or mechanical power in rotor $\quad P_m = P - I_a^2\,R_a$ —per phase

For three phases $\quad P_m = \sqrt{3}\,V_L\,I_L \cos\phi - 3\,I_a^2\,R_a$

The per phase power development in a synchronous machine is as under :

Power Flow within a Synchronous Motor

Power input/phase in stator

$$P = VI_a \cos\phi$$

Armature (*i.e.*, stator) Cu loss

$$= I_a^2 R_a$$

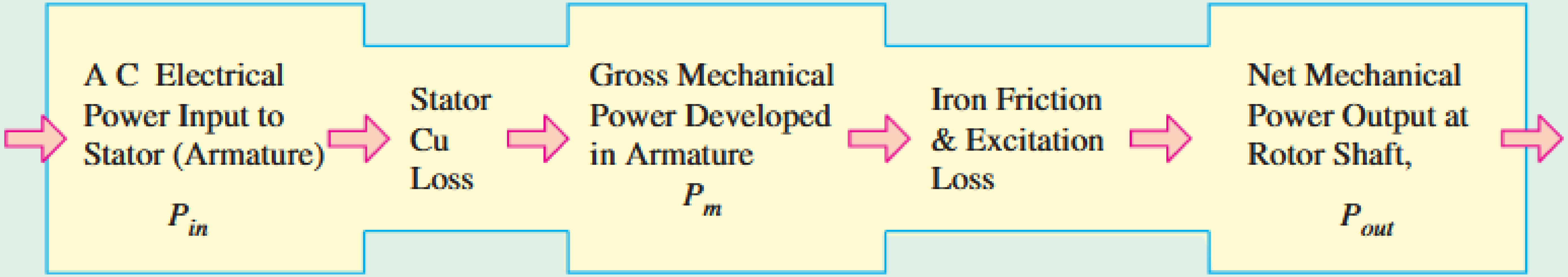
Mechanical power in armature

$$P_m = E_b I_a \cos(\alpha - \phi)$$

Iron, excitation & friction losses

Output power P_{out}

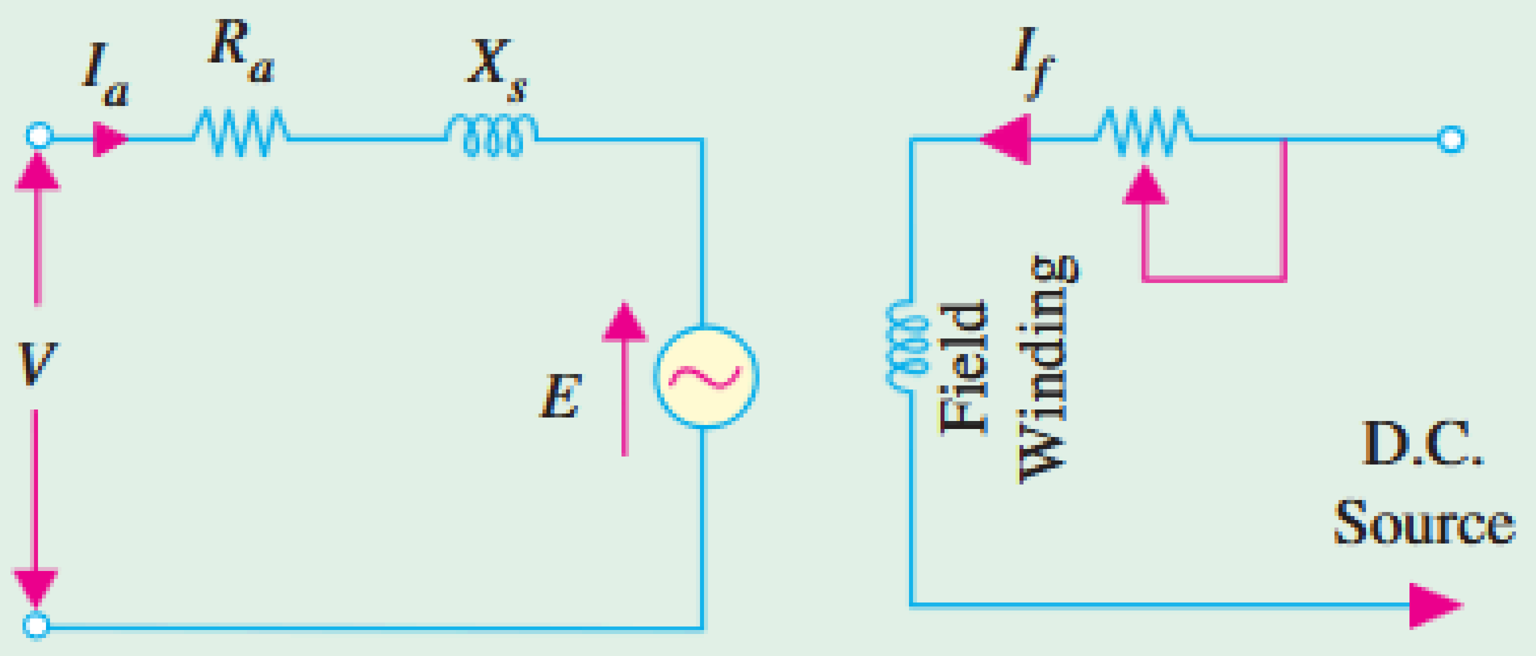
Different power stages in a synchronous motor are as under :



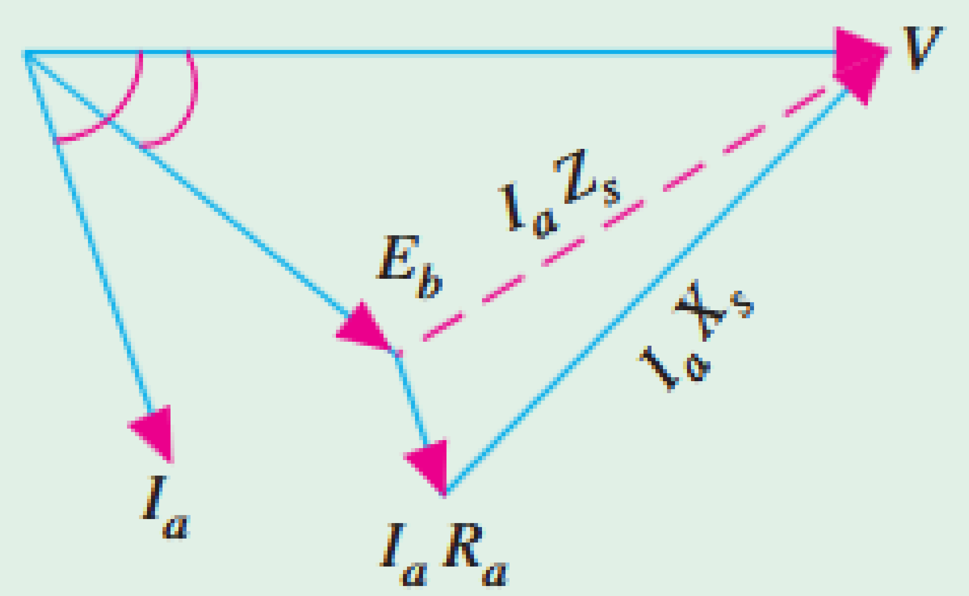
Equivalent Circuit of a Synchronous Motor

Fig. (*a*) shows the equivalent circuit model for one armature phase of a cylindrical rotor synchronous motor.

It is seen from Fig. (*b*) that the phase applied voltage V is the vector sum of reversed back e.m.f. *i.e.*, $-E_b$ and the impedance drop $I_a Z_S$. In other words, $V = (-E_b + I_a Z_S)$. The angle α* between the phasor for V and E_b is called the load angle or power angle of the synchronous motor.



(*a*)



(*b*)

Power Developed by a Synchronous Motor

Except for very small machines, the armature resistance of a synchronous motor is negligible as compared to its synchronous reactance. Hence, the equivalent circuit for the motor becomes as shown in Fig. (*a*). From the phasor diagram of Fig. (*b*), it is seen that

$$AB = E_b \sin\alpha = I_a X_S \cos\phi$$

$$\text{or} \quad VI_a \cos\phi = \frac{E_b V}{X_S} \sin\alpha$$

$$\text{Now, } VI_a \cos\phi = \text{motor power input/phase}$$

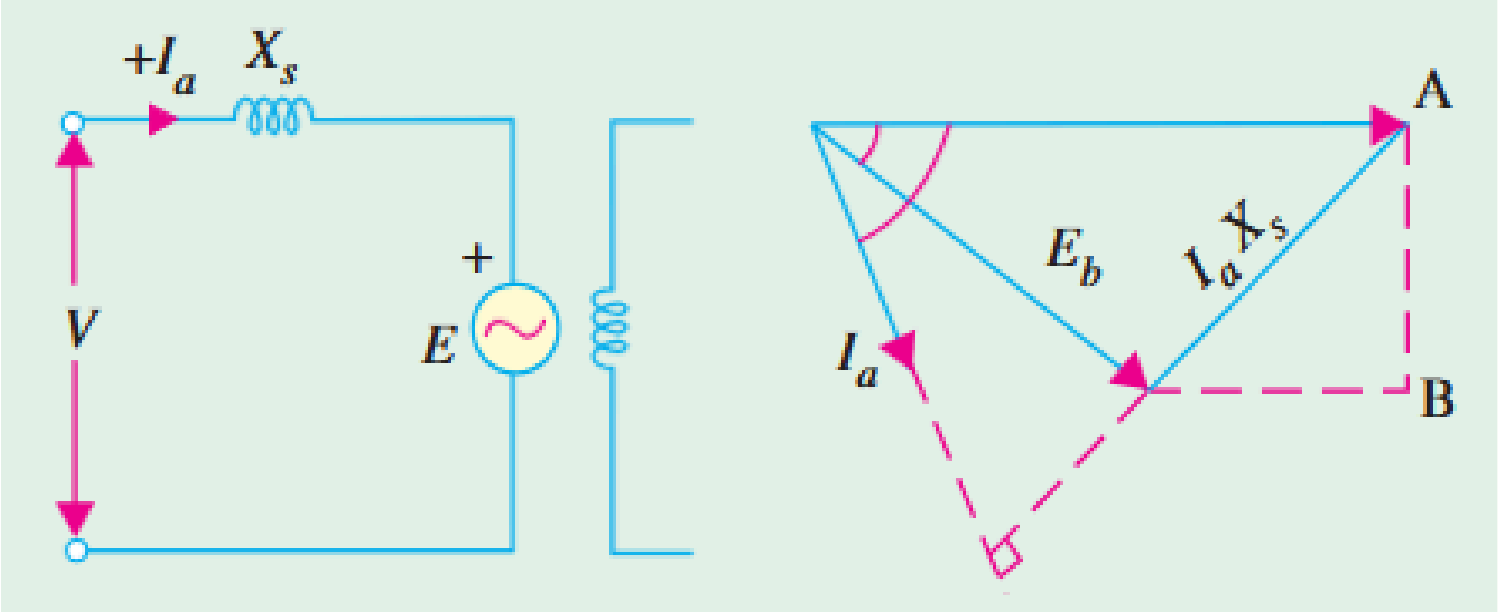
$$\therefore \quad P_{in} = \frac{E_b V}{X_S} \sin\alpha \qquad \text{...per phase*}$$

$$= 3\frac{E_b V}{X_S} \sin\alpha \qquad \text{... for three phases}$$

Since stator Cu losses have been neglected, P_{in} also represents the gross mechanical power $\{P_m\}$ developed by the motor.

$$\therefore \quad P_m = \frac{3E_b V}{X_S} \sin\alpha$$

The gross torque developed by the motor is $T_g = 9.55\, P_m / N_s$ N–m ...Ns in rpm.



(*a*) (*b*)

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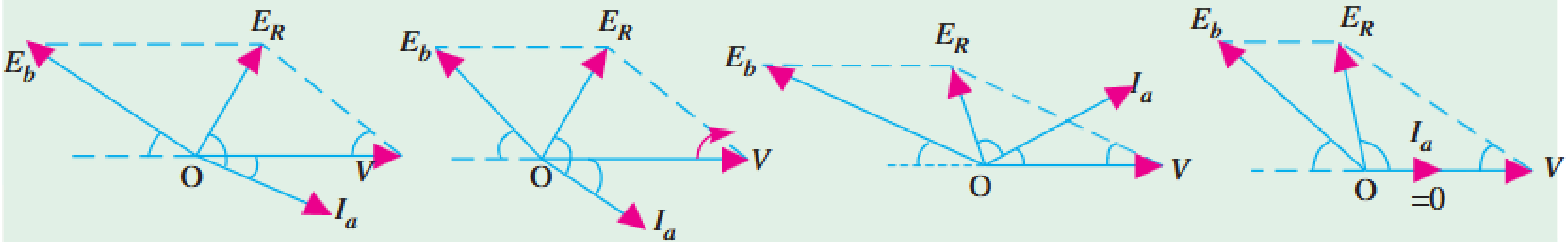
Synchronous Motor with Different Excitations

A synchronous motor is said to have normal excitation when its $E_b = V$. If field excitation is such that $E_b < V$, the motor is said to be ***under-excited***. In both these conditions, it has a lagging power factor as shown in Fig.

On the other hand, if d.c. field excitation is such that $E_b > V$, then motor is said to be ***over-excited*** and draws a leading current, as shown in Fig. (*a*). There will be some value of excitation for which armature current will be in phase with V, so that power factor will become unity, as shown in Fig. (*b*).

The value of α and back e.m.f. E_b can be found with the help of vector diagrams for various power factors, shown in Fig.



$E_b = V$ Lagging PF (*a*)

$E_b < V$ Lagging PF (*b*)

$E_b > V$ Lagging PF (*a*)

$E_b > V$ Unity PF (*b*)

Synchronous Motor with Different Excitations

(i) Lagging p.f. As seen from Fig.

$$AC^2 = AB^2 + BC^2 = [V - E_R \cos(\theta - \phi)]^2 + [E_R \sin(\theta - \phi)]^2$$

$$\therefore \quad E_b = \sqrt{[V - I_a Z_S \cos(\theta - \phi)]^2 + [I_a Z_S \sin(\theta - \phi)]^2}$$

Load angle $\alpha = \tan^{-1}\left(\frac{BC}{AB}\right) = \tan^{-1}\left[\frac{I_a Z_S \sin(\theta - \phi)}{V - I_a Z_S \cos(\theta - \phi)}\right]$

(ii) Leading p.f.

$$E_b = V + I_a Z_S \cos[180° - (\theta + \phi)] + j\, I_a Z_S \sin[180° - (\theta + \phi)]$$

$$\alpha = \tan^{-1}$$

(iii) Unity p.f.

Here, $OB = I_a R_a$ and $BC = I_a X_S$

$$\therefore \quad E_b = (V - I_a R_a) + j\, I_a X_S;\ \alpha = \tan^{-1}$$

Different Torques of a Synchronous Motor

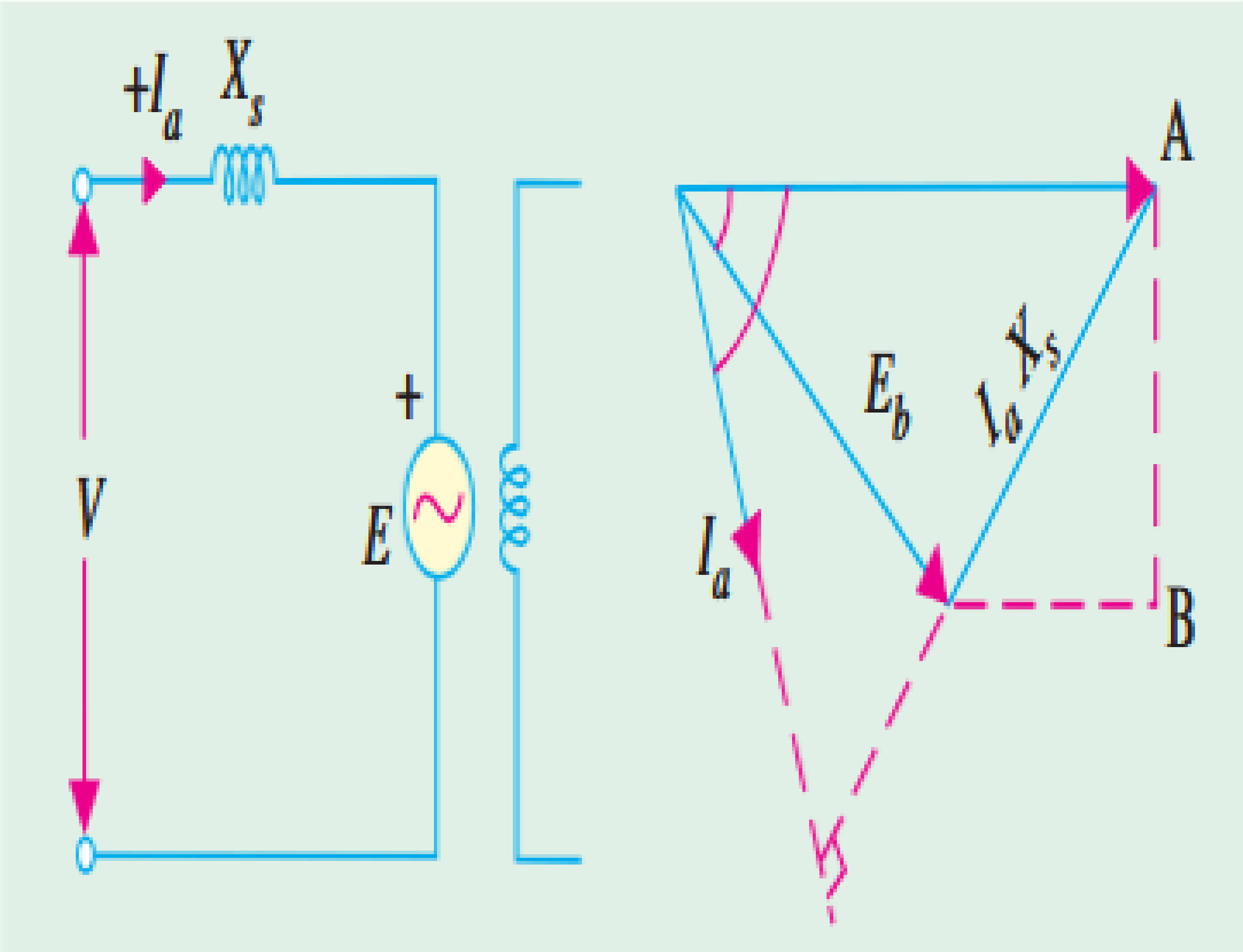
Various torques associated with a synchronous motor are as follows:

1. starting torque
2. running torque
3. pull-in torque and
4. pull-out torque

Torque motors are designed to privide maximum torque at locked rotor or near stalled conditions

Power Developed by a Synchronous Motor

In Fig. OA represents supply voltage/phase and $I_a = I$ is the armature current, AB is back e.m.f. at a load angle of α. OB gives the resultant voltage $E_R = IZ_S$ (or IX_S if R_a is negligible). I leads V by ϕ and lags behind E_R by an angle $\theta = \tan^{-1}(X_S / R_a)$. Line CD is drawn at an angle of θ to AB. AC and ED are $\perp$ to CD (and hence to AE also).



(a) (b)

Mechanical power per phase developed in the rotor is

$$P_m = E_b I \cos\psi \qquad ...(i)$$

In $\Delta\, OBD$, $BD = I Z_S \cos\psi$

Now, $BD = CD - BC = AE - BC$

$$I Z_S \cos\psi = V \cos(\theta - \alpha) - E_b \cos\theta$$

$$\therefore \quad I\cos\psi = \frac{V}{Z_S}\cos(\theta-\alpha) - \frac{E_b}{Z_S}\cos\theta$$

Substituting this value in (i), we get

$$P_m \text{ per phase} = E_b\left[\frac{V}{Z_S}\cos(\theta-\alpha) - \frac{E_b}{Z_S}\cos\theta\right] = \frac{E_b V}{Z_S}\cos(\theta-\alpha) - \frac{E_b^2}{Z_S}\cos\theta \text{ *} \qquad ...(ii)$$

Power Developed by a Synchronous Motor

This is the expression for the mechanical power developed in terms of the load angle (α) and the internal angle (θ) of the motor for a constant voltage V and E_b (or excitation because E_b depends on excitation only).

If T_g is the gross armature torque developed by the motor, then

$$T_g \times 2\pi N_S = P_m \text{ or } T_g = P_m/\omega_s = P_m / 2\pi N_S \qquad -N_S \text{ in rps}$$

$$T_g = \frac{P_m}{2\pi N_S/60} = \frac{60}{2\pi}\cdot\frac{P_m}{N_S} = 9.55\frac{P_m}{N_S} \qquad -N_S \text{ in rpm}$$

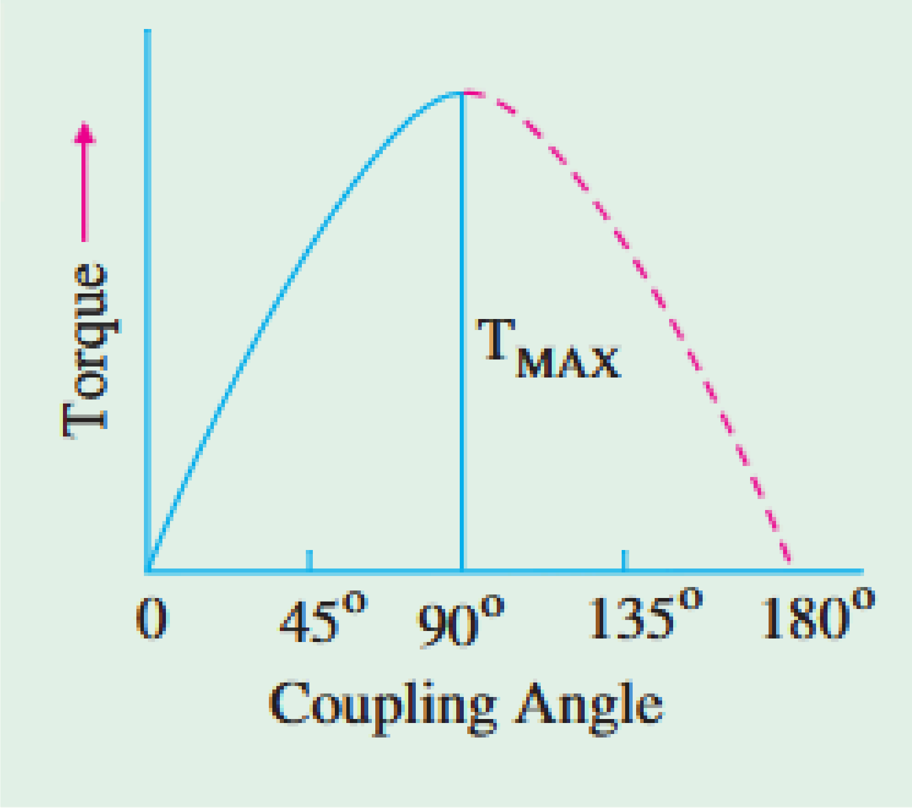
Condition for maximum power developed can be found by differentiating the above expression with respect to load angle and then equating it to zero.

$$\therefore \quad \frac{dP_m}{d\alpha} = -\frac{E_b V}{Z_S}\sin(\theta-\alpha) = 0 \quad \text{or} \quad \sin(\theta-\alpha) = 0 \qquad \therefore \quad \theta = \alpha$$

Power Developed by a Synchronous Motor

$\therefore$ value of maximum power $(P_m)_{max} = \dfrac{E_b V}{Z_S} - \dfrac{E_b}{Z_S}\cos\alpha$ or $(P_m)_{max} = \dfrac{E_b V}{Z_S} - \dfrac{E_b}{Z_S}\cos\theta$. ...(*iii*)

This shows that the maximum power and hence torque ($\because$ speed is constant) depends on V and E_b *i.e.*, excitation. Maximum value of θ (and hence α) is 90°. For all values of V and E_b, this limiting value of α is the same but maximum torque will be proportional to the maximum power developed as given in equation **(*iii*)**. Equation **(*ii*)** is plotted in Fig. 38.22.



If R_a is neglected, then $Z_S \cong X_S$ and $\theta = 90°$ $\therefore \cos\theta = 0$

$P_m = \dfrac{E_b V}{X_S}\sin\alpha$ ...(*iv*) $(P_m)_{max} = \dfrac{E_b V}{X_S}$ ... from equation **(*iii*)*** The same value can be otained by putting $\alpha = 90°$ in equation **(*iv*)**. This corresponds to the 'pull-out' torque.

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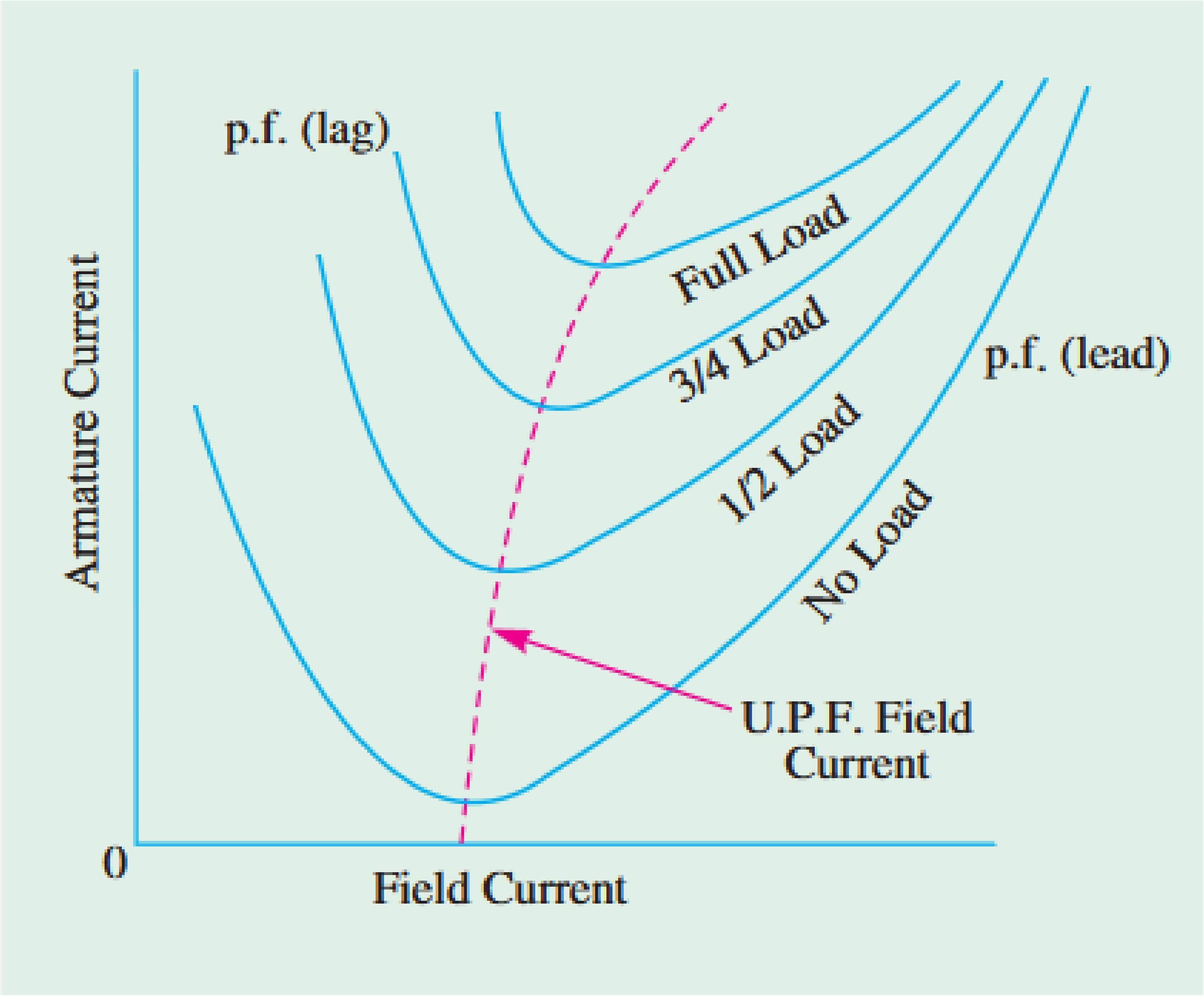
Construction of V-curves

The V-curves of a synchornous motor show how armature current varies with its field current when motor ***input is kept constant***. These are obtained by plotting a.c. armature current against d.c. field current while motor input is kept constant and are so called because of their shape (Fig. There is a family of such curves, each corresponding to a definite power intake.



In order to draw these curves experimentally, the motor is run from constant voltage and constant-frequency bus-bars. Power input to motor is kept constant at a definite value. Next, field current is increased in small steps and corresponding armature currents are noted. When plotted, we get a V-curve for a particular constant motor input. Similar curves can be drawn by keeping motor input constant at different values. A family of such curves is

Construction of V-curves

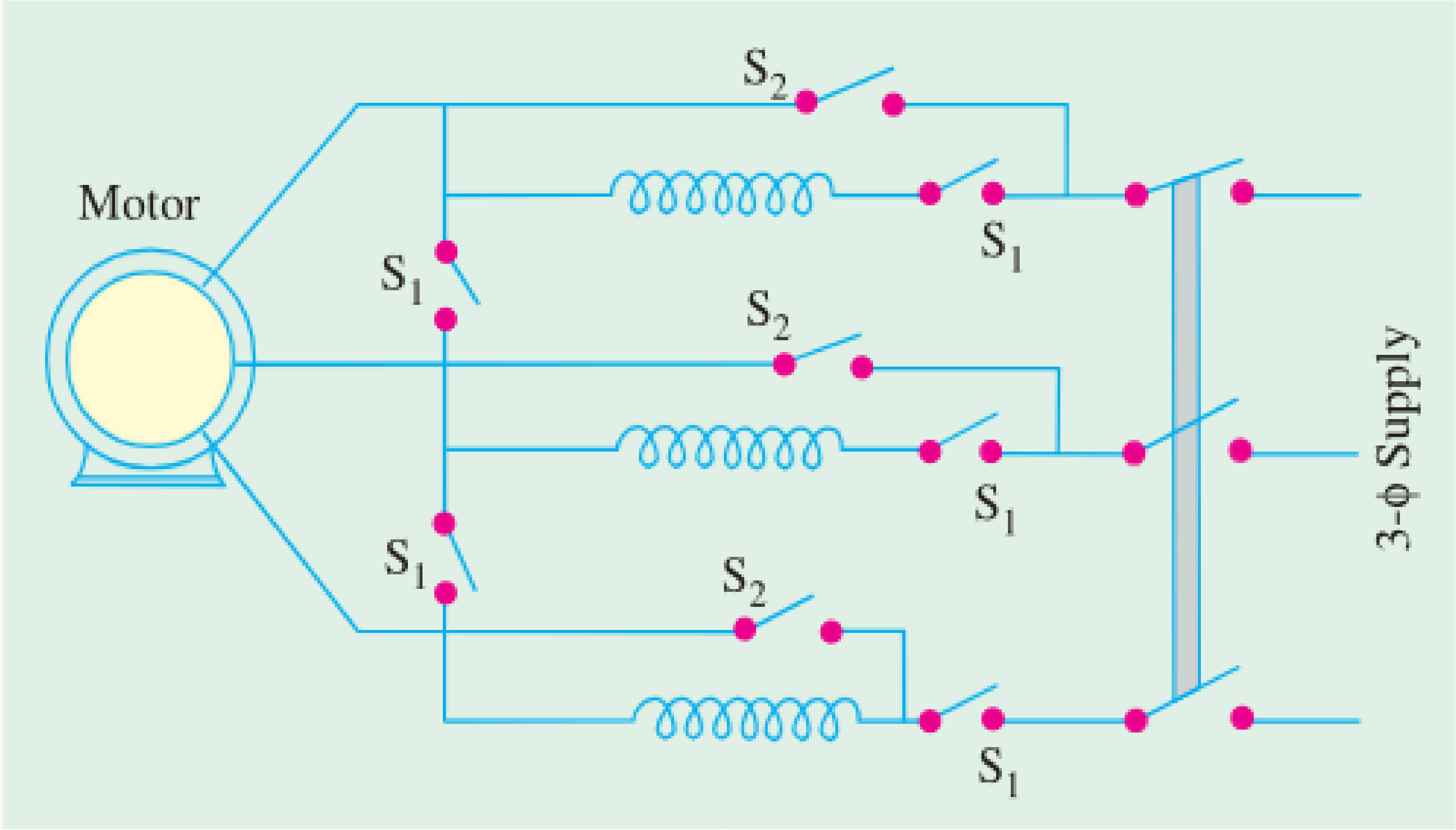
Detailed procedure for graphic construction of V-curves is given below :

1. First, constant-power lines are drawn as discussed in Art.
2. Then, with A as the centre, concentric circles of different radii AB, AB_1, AB_2, etc. are drawn where AB, AB_1, AB_2, etc., are the back e.m.fs corresponding to different excitations. The intersections of these circles with lines of constant power give positions of the working points for specific loads and excitations (hence back e.m.fs). The vectors OB, OB_1, OB_2 etc., represent different values of E_R (and hence currents) for different excitations. Back e.m.f. vectors AB, AB_1 etc., have not been drawn purposely in order to avoid confusion (Fig. 38.56).
3. The different values of back e.m.fs like AB, AB_1, AB_2, etc., are projected on the magnetisation and corresponding values of the field (or exciting) amperes are read from it.
4. The field amperes are plotted against the corresponding armature currents, giving us 'V' curves.

Methods of Starting

1. At the beginning, when voltage is applied, the rotor is stationary. The rotating field of the stator winding induces a very large e.m.f. in the rotor during the starting period, though the value of this e.m.f. goes on decreasing as the rotor gathers speed.

   Normally, the field windings are meant for 110-V (or 250 V for large machines) but during starting period there are many thousands of volts induced in them. Hence, the rotor windings have to be highly insulated for withstanding such voltages.



Methods of Starting

2. When full line voltage is switched on to the armature at rest, a very large current, usually 5 to 7 times the full-load armature current is drawn by the motor. In some cases, this may not be objectionable but where it is, the applied voltage at starting, is reduced by using auto-transformers (Fig.). However, the voltage should not be reduced to a very low value because the starting torque of an induction motor varies approximately as the square of the applied voltage. Usually, a value of 50% to 80% of the full-line voltage is satisfactory.

   Auto-transformer connections are shown in Fig. For reducing the supply voltage, the switches S_1 are closed and S_2 are kept open. When the motor has been speeded-up, S_2 are closed and S_1 opened to cut out the transformers.

Procedure for Starting a Synchronous Motor

1. First, main field winding is short-circuited.
2. Reduced voltage with the help of auto-transformers is applied across stator terminals. The motor starts up.
3. When it reaches a steady speed (as judged by its sound), a weak d.c. excitation is applied by removing the short-circuit on the main field winding. If excitation is sufficient, then the machine will be pulled into synchronism.
4. Full supply voltage is applied across stator terminals by cutting out the auto-transformers.
5. The motor may be operated at any desired power factor by changing the d.c. excitation.

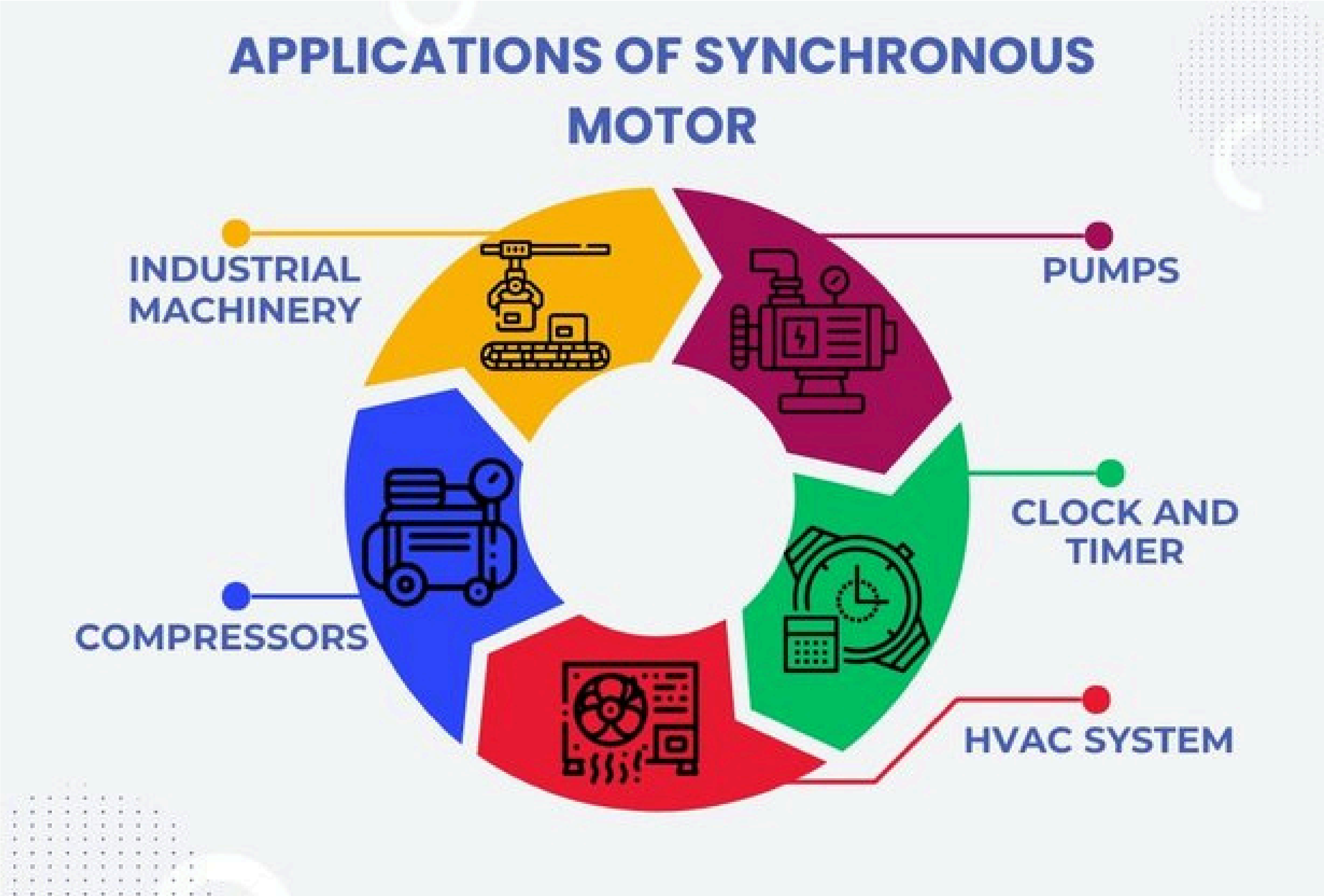
Comparison Between Synchronous and Induction Motors

1. For a given frequency, the synchronous motor runs at a constant average speed whatever the load, while the speed of an induction motor falls somewhat with increase in load.
2. The synchronous motor can be operated over a wide range of power factors, both lagging and leading, but induction motor always runs with a lagging p.f. which may become very low at light loads.
3. A synchronous motor is inherently not self-starting.
4. The changes in applied voltage do not affect synchronous motor torque as much as they affect the induction motor torque. The breakdown torque of a synchronous motor varies approximately as the first power of applied voltage whereas that of an induction motor depends on the square of this voltage.
5. A d.c. excitation is required by synchronous motor but not by induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors, but they are particularly attractive for low-speed drives (below 300 r.p.m.) because their power factor can always be adjusted to 1.0 and their efficiency is high. However, induction motors are excellent for speeds above 600 r.p.m.
7. Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Such motors of 10 MW range are used for driving crushers, rotary kilns and variable-speed ball mills etc.

Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service :

1. Power factor correction
2. Constant-speed, constant-load drives
3. Voltage regulation



WEEK 12

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Mathematical Problems on Synchronous Motor

Mathematical problems related to transformers will be practiced and solved during classroom sessions. Problems from the prescribed reference book will be addressed, and additional practice materials will be provided to enhance understanding and proficiency.

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Induction Motor

As a general rule, conversion of electrical power into mechanical power takes place in the ***rotating*** part of an electric motor. In d.c. motors, the electric power is ***conducted*** directly to the armature (*i.e.* rotating part) through brushes and commutator Hence, in this sense, a d.c. motor can be called a ***conduction*** motor. However, in a.c. motors, the rotor does not receive electric power by conduction but by ***induction*** in exactly the same way as the secondary of a 2-winding transformer receives its power from the primary. That is why such motors are known as ***induction*** motors. In fact, an induction motor can be treated as a ***rotating transformer*** *i.e.* one in which primary winding is stationary but the secondary is free to rotate

Of all the a.c. motors, the polyphase induction motor is the one which is extensively used for various kinds of industrial drives. It has the following main advantages and also some dis-advantages:

Induction Motor

Advantages:

1. It has very simple and extremely rugged, almost unbreakable construction (especially squirrel-cage type).
2. Its cost is low and it is very reliable.
3. It has sufficiently high efficiency. In normal running condition, no brushes are needed, hence frictional losses are reduced. It has a reasonably good power factor.
4. It requires minimum of maintenance.
5. It starts up from rest and needs no extra starting motor and has not to be synchronised. Its starting arrangement is simple especially for squirrel-cage type motor.

Disadvantages:

1. Its speed cannot be varied without sacrificing some of its efficiency.
2. Just like a d.c. shunt motor, its speed decreases with increase in load.
3. Its starting torque is somewhat inferior to that of a d.c. shunt motor.

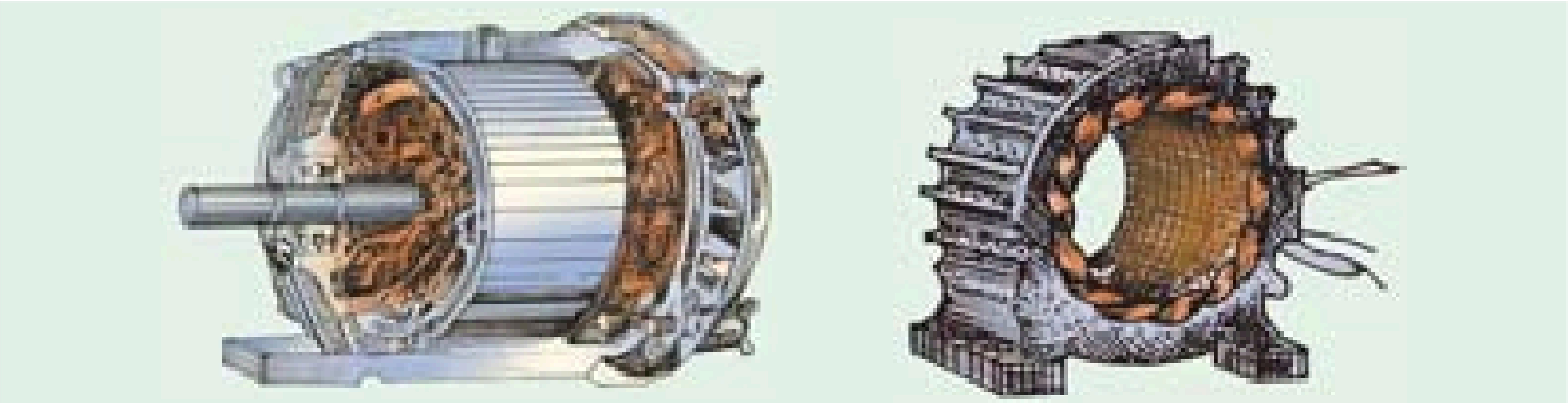
Construction

An induction motor consists essentially of two main parts :

(*a*) a stator and (*b*) a rotor.

(*a*) Stator

The stator of an induction motor is, in principle, the same as that of a synchronous motor or generator. It is made up of a number of stampings, which are slotted to receive the windings [Fig. (*a*)]. The stator carries a 3-phase winding [Fig. (*b*)] and is fed from a 3-phase supply. It is wound for a definite number of poles*, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and *vice versa*. It will be shown in that the stator windings, when supplied with 3-phase currents, produce a magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed (given by $N_s = 120\, f/P$). This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.



Construction

(*b*) Rotor

(*i*) ***Squirrel-cage rotor* :** Motors employing this type of rotor are known as squirrel-cage induction motors.

(*ii*) ***Phase-wound or wound rotor* :** Motors employing this type of rotor are variously known as 'phase-wound' motors or 'wound' motors or as 'slip-ring' motors.

Squirrel-cage Rotor

Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which, it should be

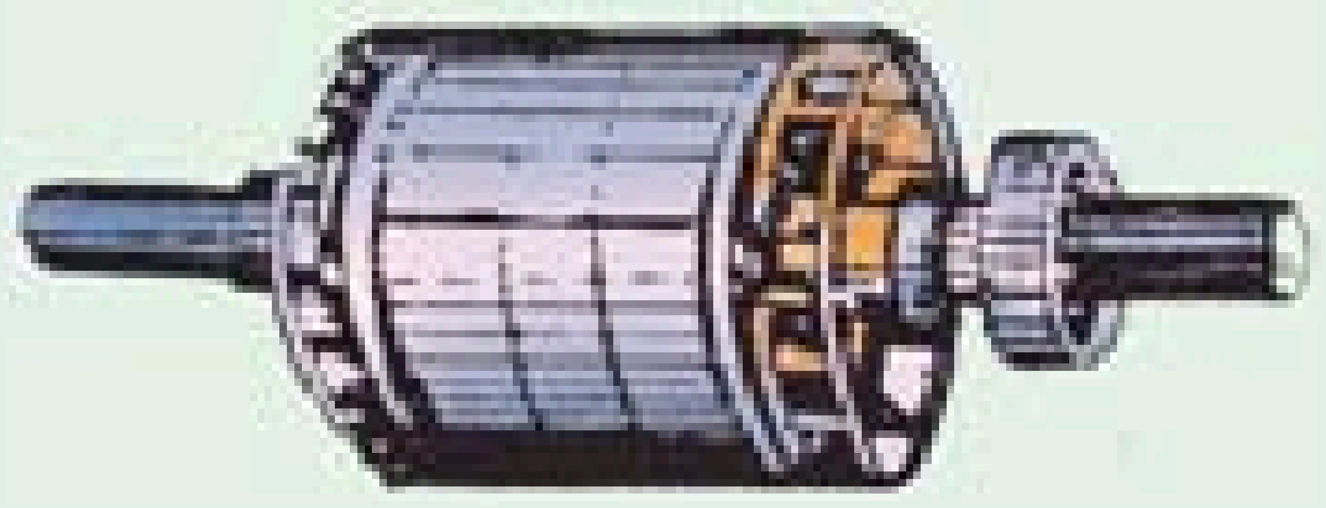


Fig. (*a*) Squirrel-cage rotor with copper bars and alloy brazed end-rings (*Courtesy : Gautam Electric Motors*)

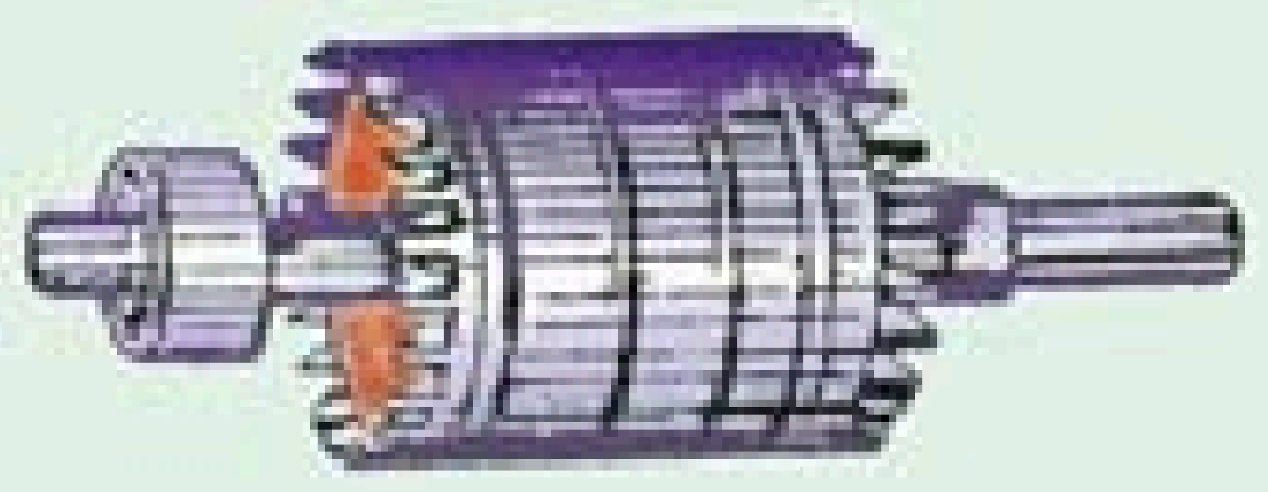


Fig. (*b*) Rotor with shaft and brings (*Courtesy : Gautam Electric Motors*)

noted clearly, are not wires but consist of heavy bars of copper, aluminium or alloys. One bar is placed in each slot, rather the bars are inserted from the end when semi-closed slots are used. The rotor bars are brazed or electrically welded or bolted to two heavy and stout short-circuiting end-rings, thus giving us, what is so picturesquely called, a squirrel-case construction (Fig.).

It should be noted that the ***rotor bars are permanently short-circuited on themselves***, hence it is not possible to add any external resistance in series with the rotor circuit for starting purposes.

Phase-wound Rotor

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase even ***when the stator is wound two-phase***.

The three phases are starred internally. The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them [Fig. (*b*)]. These three brushes are further externally connected to a 3-phase star-connected rheostat [Fig. (*c*)]. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor, as shown in Fig. (*a*) (Ex.. and) and for changing its speed-torque/current characteristics. When running under normal conditions, the ***slip-rings are automatically short-circuited*** by means of a metal collar, which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and the wear and tear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like the squirrel-case rotor.

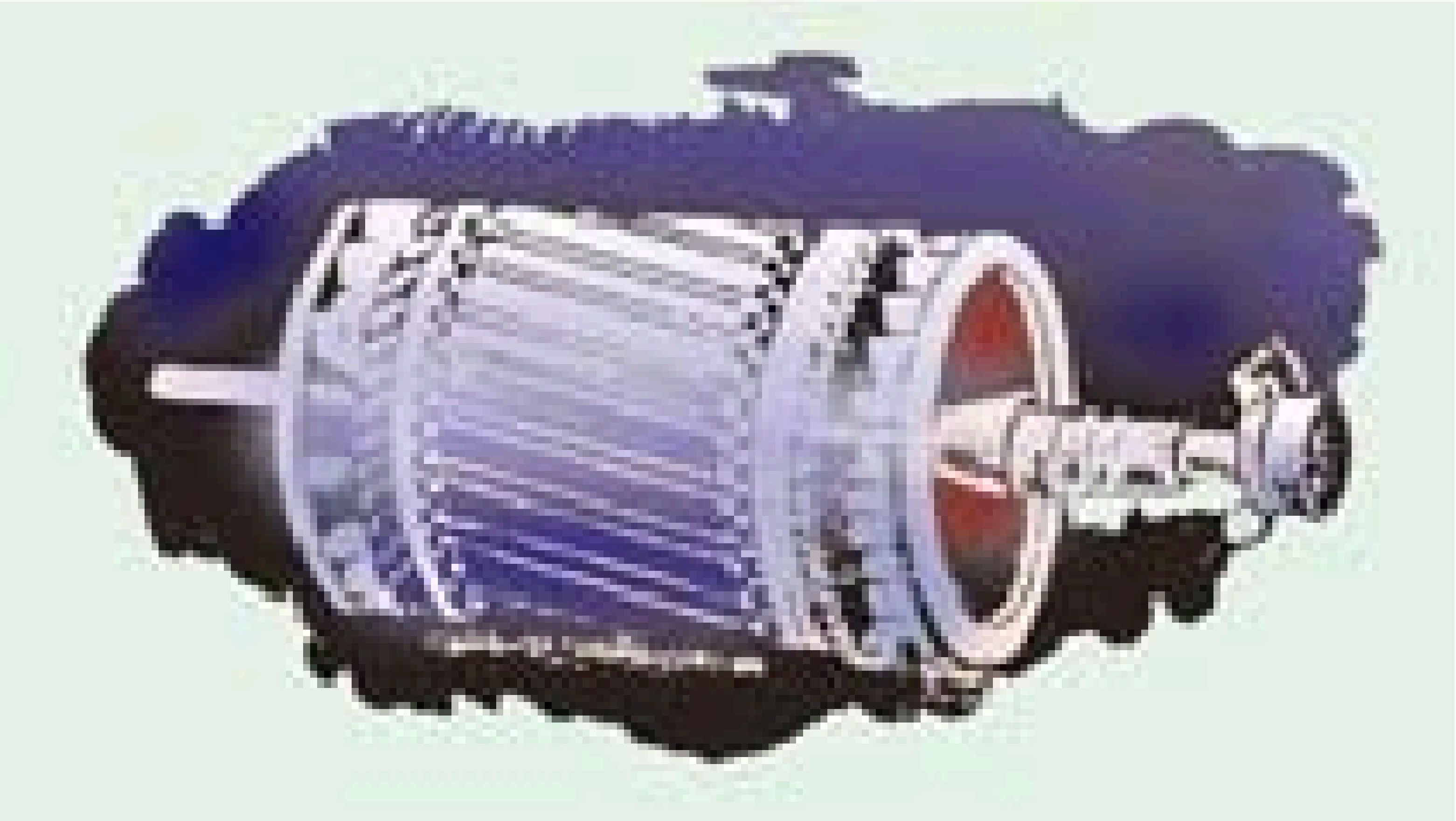


Fig. 34.5 (*a*)

Phase-wound Rotor

1. **Frame.** Made of close-grained alloy cast iron.
2. **Stator and Rotor Core.** Built from high-quality low-loss silicon steel laminations and flash-enamelled on both sides.
3. **Stator and Rotor Windings.** Have moisture proof tropical insulation embodying mica and high quality varnishes. Are carefully spaced for most effective air circulation and are rigidly braced to withstand centrifugal forces and any short-circuit stresses.
4. **Air-gap.** The stator rabbets and bore are machined carefully to ensure uniformity of air-gap.
5. **Shafts and Bearings.** Ball and roller bearings are used to suit heavy duty, toruble-free running and for enhanced service life.
6. **Fans.** Light aluminium fans are used for adequate circulation of cooling air and are securely keyed onto the rotor shaft.
7. **Slip-rings and Slip-ring Enclosures.** Slip-rings are made of high quality phosphor-bronze and are of moulded construction.

WEEK 14

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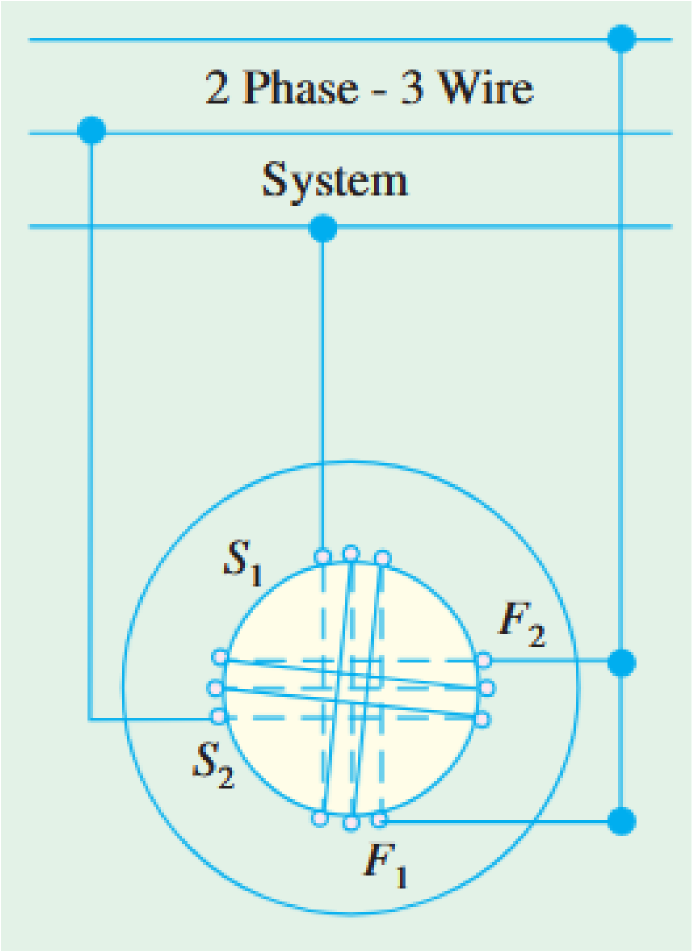
Production of Rotating Field



(*a*) When $\theta = 0°$ *i.e.* corresponding to point 0 in Fig. $\Phi_1 = 0$, but Φ_2 is maximum *i.e.* equal to Φ_m and negative. Hence, resultant flux $\Phi_r = \Phi_m$ and, being negative, is shown by a vector pointing downwards [Fig (*i*)].

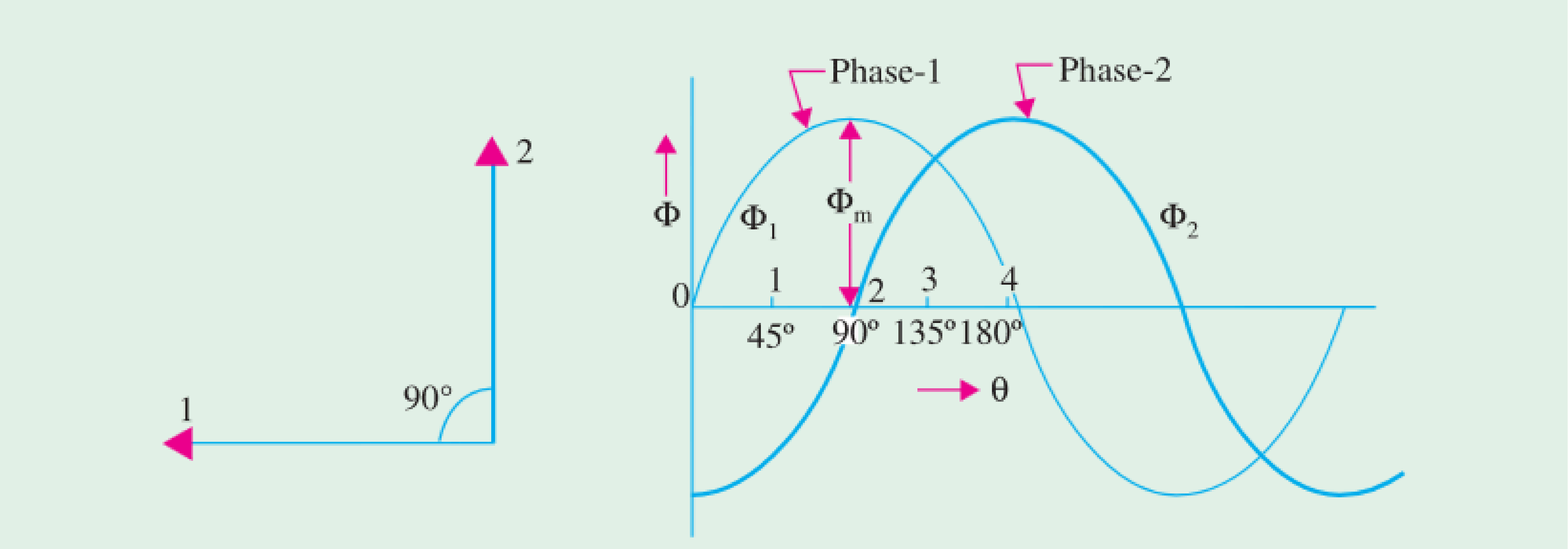
(*b*) When $\theta = 45°$ *i.e.* corresponding to point 1 in Fig. . At this instant, $\Phi_1 = \Phi_m/\sqrt{2}$ and is positive; $\Phi_2 = \Phi_m/\sqrt{2}$ but is still negative. Their resultant, as shown in Fig (*ii*), is $\Phi_r =$

$\sqrt{[(\phi_m/\sqrt{2})^2 + (\phi_m/\sqrt{2})^2]} = \Phi_m$ although this resultant has shifted 45° clockwise.

(*c*) When $\theta = 90°$ *i.e.* corresponding to point 2 in Fig. Here $\Phi_2 = 0$, but $\Phi_1 = \Phi_m$ and is positive. Hence, $\Phi_r = \Phi_m$ and has further shifted by an angle of 45° from its position in (*b*) or by 90° from its original position in (*a*).

(*d*) When $\theta = 135°$ *i.e.* corresponding to point 3 in Fig. . Here, $\Phi_1 = \Phi_m/\sqrt{2}$ and is positive, $\Phi_2 = \Phi_m/\sqrt{2}$ and is also positive. The resultant $\Phi_r = \Phi_m$ and has further shifted clockwise by another 45°, as shown in Fig. (*iv*).

Production of Rotating Field



Why Does the Rotor Rotate ?

The rotor in an electrical machine rotates due to the interaction between magnetic fields and current-carrying conductors, which produces a torque. Here's an explanation of the process:

Magnetic Field Creation: When an electrical current flows through the stator windings (stationary part of the machine), it creates a rotating magnetic field in the air gap between the stator and rotor.

Induced Currents (in Induction Motors): If the rotor is not rotating initially, the relative motion between the rotor and the rotating magnetic field induces currents in the rotor conductors, following Faraday's Law of Electromagnetic Induction.

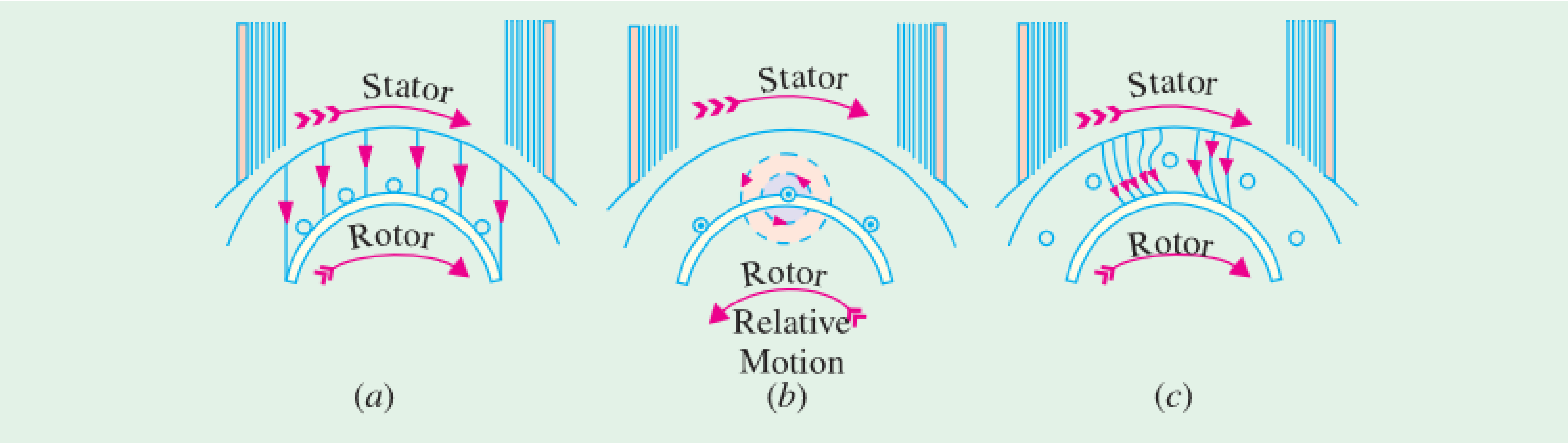
Lorentz Force: The interaction between the induced currents in the rotor and the stator's magnetic field generates a force (Lorentz force) on the rotor conductors. This force acts tangentially to the rotor, creating a torque.

Synchronization in Synchronous Motors: In synchronous motors, the rotor already contains a permanent magnet or an electromagnet. The rotor aligns itself with the stator's rotating magnetic field, causing continuous rotation.

Energy Conversion: The torque produced by the interaction of the magnetic fields and current causes the rotor to turn, converting electrical energy into mechanical energy.

In summary, the rotor rotates due to the magnetic interaction between the stator and rotor fields. This principle is foundational to the operation of various electrical machines, such as motors and generators.

Why Does the Rotor Rotate ?



Slip

In practice, the rotor never succeeds in 'catching up' with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speeds depends upon the load on the motor.*

The difference between the synchronous speed N_s and the actual speed N of the rotor is known as ***slip***. Though it may be expressed in so many revolutions/second, yet it is usual to express it as a percentage of the synchronous speed. Actually, the term '***slip***' is descriptive of the way in which the rotor 'slips back' from synchronism.

$$\% \text{ slip} \quad s = \frac{N_s - N}{N_s} \times 100$$

Sometimes, $N_s - N$ is called the ***slip speed.***

Obviously, rotor (or motor) speed is $N = N_s (1 - s)$.

It may be kept in mind that revolving flux is rotating synchronously, relative to the stator (*i.e.* stationary space) but at slip speed relative to the rotor.

Relation Between Torque and Rotor Power Factor

it has been shown that in the case of a d.c. motor, the torque T_a is proportional to the product of armature current and flux per pole *i.e.* $T_a \propto \phi I_a$. Similarly, in the case of an induction motor, the torque is also proportional to the product of flux per stator pole and the rotor current. However, there is one more factor that has to be taken into account *i.e.* the power factor of the rotor.

$\therefore \quad T \propto \phi I_2 \cos \phi_2 \quad$ or $\quad T = k \phi I_2 \cos \phi_2$

where I_2 = rotor current at standstill

ϕ_2 = angle between rotor e.m.f. and rotor current

k = a constant

Denoting rotor e.m.f. at ***standstill*** by E_2, we have that $E_2 \propto \phi$

$\therefore \quad T \propto E_2 I_2 \cos \phi_2$

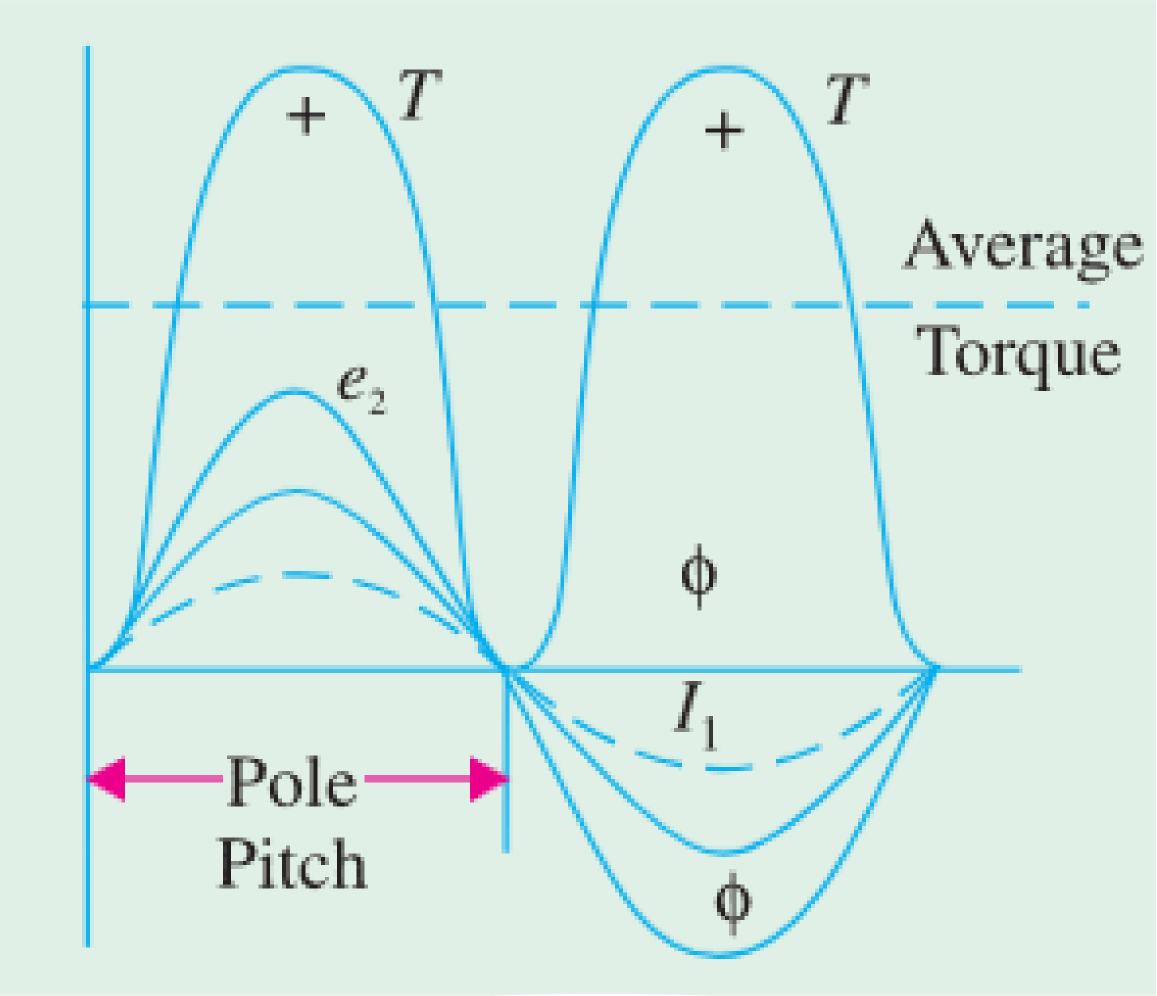
Relation Between Torque and Rotor Power Factor

or $$T = k_1 E_2 I_2 \cos \phi_2$$

where k_1 is another constant.

The effect of rotor power factor on rotor torque is illustrated in Fig. and Fig. for various values of ϕ_2. From the above expression for torque, it is clear that as ϕ_2 increases (and hence, $\cos \phi_2$ decreases) the torque decreases and ***vice versa***.

In the discussion to follow, the stator flux distribution is assumed sinusoidal. This revolving flux induces in each rotor conductor or bar an *e.m.f.* whose value depends on the flux density, in which the conductor is lying at the instant considered ($\because$ $e = Blv$ volt). Hence, the induced e.m.f. in the rotor is also sinusoidal.



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Starting Torque

The torque developed by the motor at the instant of starting is called starting torque. In some cases, it is greater than the normal running torque, whereas in some other cases it is somewhat less.

Let E_2 = rotor *e.m.f.* per phase at ***standstill***;

R_2 = rotor resistance/phase

X_2 = rotor reactance/phase at ***standstill***

$\therefore \quad Z_2 = \sqrt{(R_2^2 + X_2^2)}$ = rotor impedance/phase at ***standstill***

Then, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{E_2}{\sqrt{(R_2^2 + X_2^2)}}; \quad \cos\phi_2 = \dfrac{R_2}{Z_2} = \dfrac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$

Standstill or starting torque $T_{st} = k_1 E_2 I_2 \cos\phi_2$

or $$T_{st} = k_1 E_2 \cdot \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \quad \text{...}(i)$$

If supply voltage V is constant, then the flux Φ and hence, E_2 both are constant.

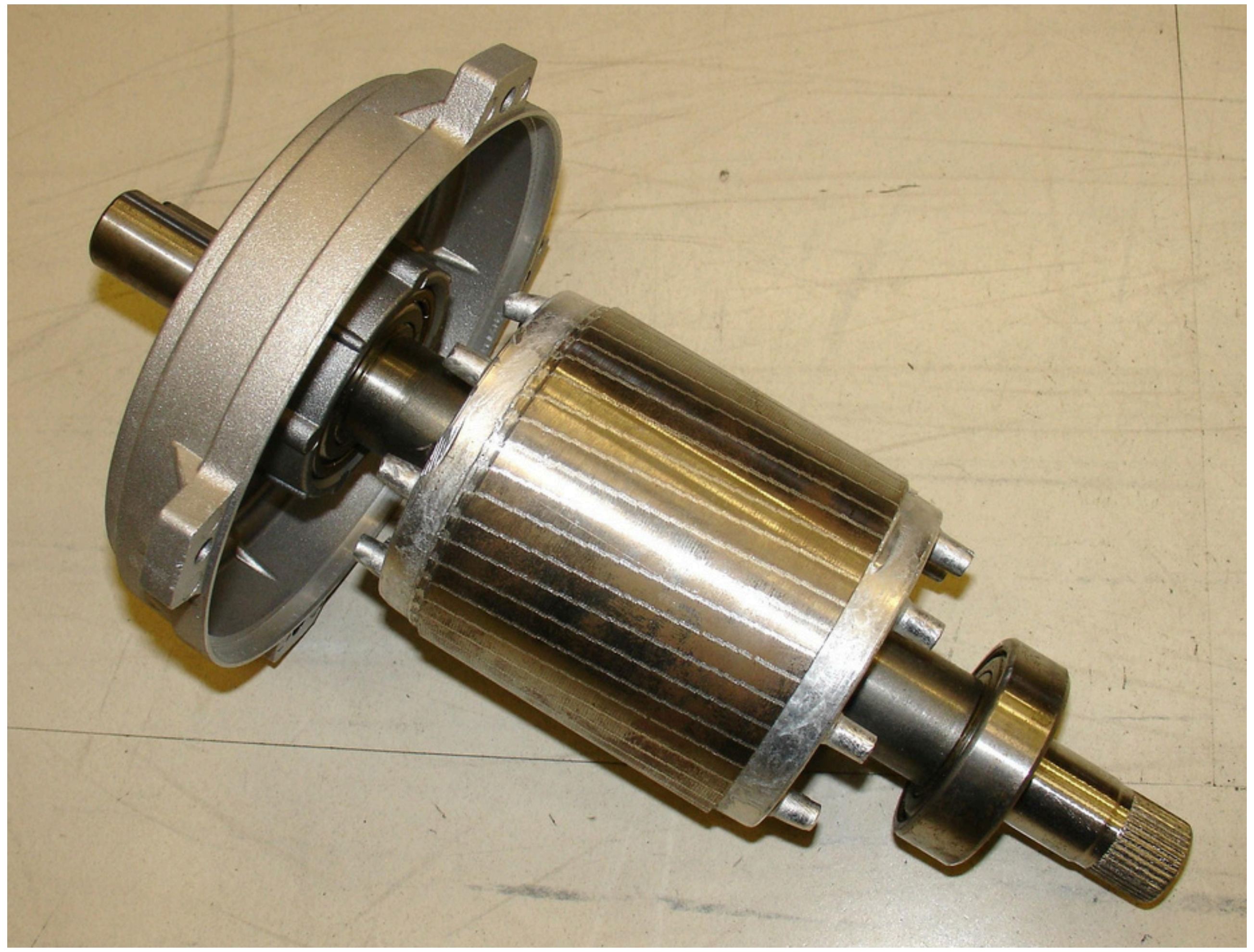
$\therefore \quad T_{st} = k_2 \dfrac{R_2}{R_2^2 + X_2^2} = k_2 \dfrac{R_2}{Z_2^2}$ where k_2 is some other constant.

Now, $k_1 = \dfrac{3}{2\pi N_s}$, $\quad \therefore \quad \boxed{T_{st} = \dfrac{3}{2\pi N_s} \cdot \dfrac{E_2^2 R_2}{R_2^2 + X_2^2}}$

Where Ns $\rightarrow$ synchronous speed in rps.

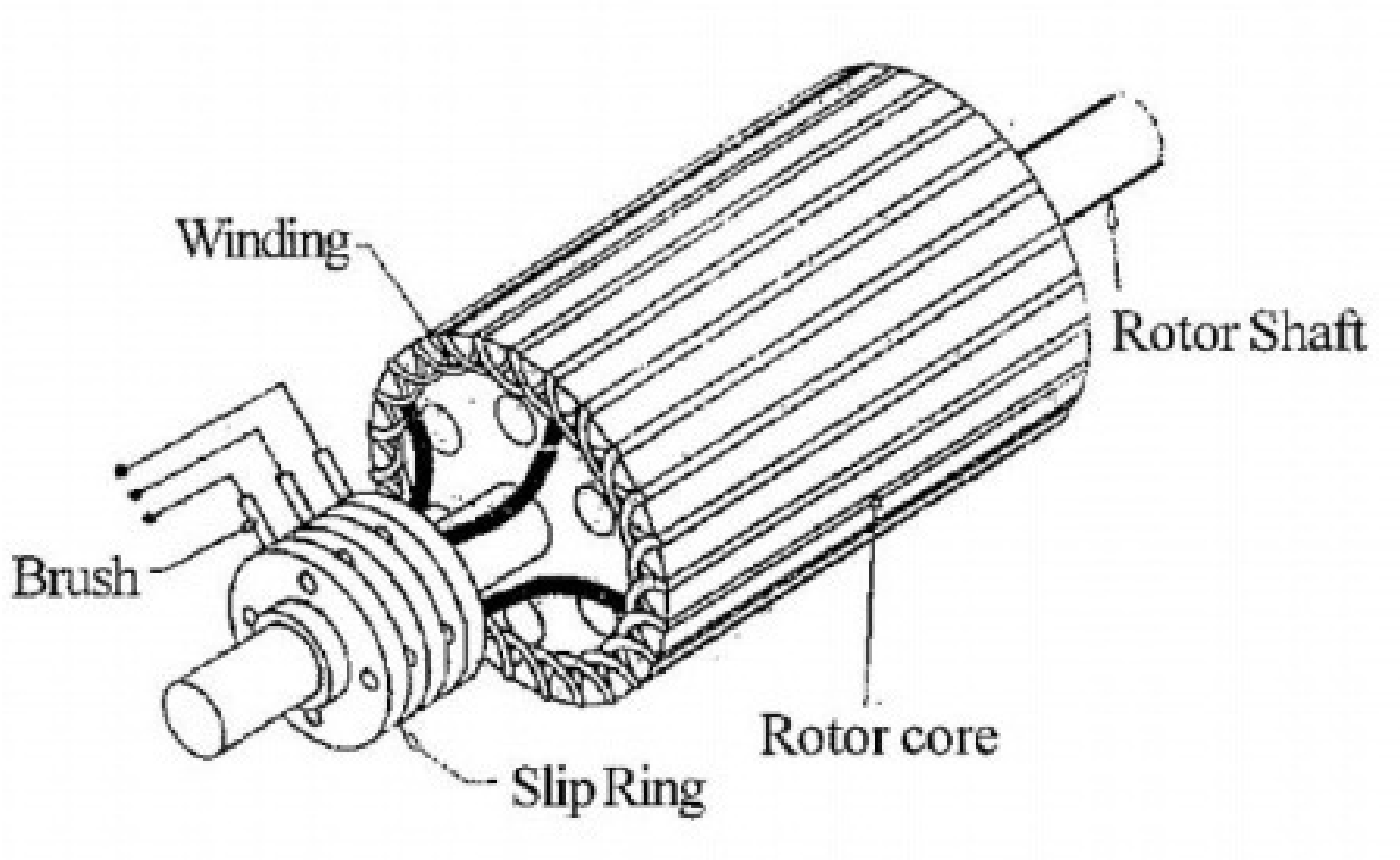
Starting Torque of a Squirrel Cage Motor

The resistance of a squirrel-cage motor is fixed and small as compared to its reactance which is very large especially at the start because at standstill, the frequency of the rotor currents equals the supply frequency. Hence, the starting current I_2 of the rotor, though very large in magnitude, lags by a very large angle behind E_2, with the result that the starting torque per ampere is very poor. It is roughly 1.5 times the full-load torque, although the starting current is 5 to 7 times the full-load current. Hence, such motors are not useful where the motor has to start against heavy loads.



Starting Torque of a Slip Ring Motor

The starting torque of such a motor is increased by improving its power factor by adding external resistance in the rotor circuit from the star-connected rheostat, the rheostat resistance being progressively cut out as the motor gathers speed. Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current. At first, the effect of improved power factor predominates the current-decreasing effect of impedance. Hence, starting torque is increased. But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.



Condition for Maximum Starting Torque

It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

Now $$T_{st} = \frac{k_2 R_2}{R_2^2 + X_2^2}$$

$$\therefore \quad \frac{dT_{st}}{dR_2} = k_2 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2 \, (2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

or $$R_2^{\;2} + X_2^{\;2} = 2R_2^{\;2}$$

$$\therefore \quad R_2 = X_2.$$

Effect of Change in Supply Voltage on Starting Torque

We have seen in that $T_{st} = \dfrac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$. Now $E_2 \propto$ supply voltage V

$$\therefore \qquad \boxed{T_{st} = \frac{k_3 V^2 R_2}{R_2^2 + X_2^2} = \frac{k_3 V^2 R_2}{Z_2^2}}$$

where k_3 is yet another constant. Hence $T_{st} \propto V^2$.

Clearly, the torque is very sensitive to any changes in the supply voltage. A change of 5 per cent in supply voltage, for example, will produce a change of approximately 10% in the rotor torque. This fact is of importance in star-delta and auto transformer starters

Torque Under Running Conditions

$$T \propto E_r I_r \cos\phi_2 \text{ or } T \propto \phi\, I_r \cos\phi_2 \qquad (\because E_r \propto \phi)$$

where E_r = rotor e.m.f./phase under ***running conditions***

I_r = rotor current/phase under ***running conditions***

Now $E_r = sE_2$

$$\therefore \quad I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\cos\phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \quad \text{—Fig. 34.20}$$

$$\therefore \quad T \propto \frac{s\,\Phi\,E_2R_2}{R_2^2 + (sX_2)^2} = \frac{k\Phi\,.\,s\,.\,E_2R_2}{R_2^2 + (sX_2)^2}$$

$$\text{Also } T = \frac{k_1\,.\,sE_2^2\,R_2}{R_2^2 + (sX_2)^2} \quad (\because E_2 \propto \phi)$$

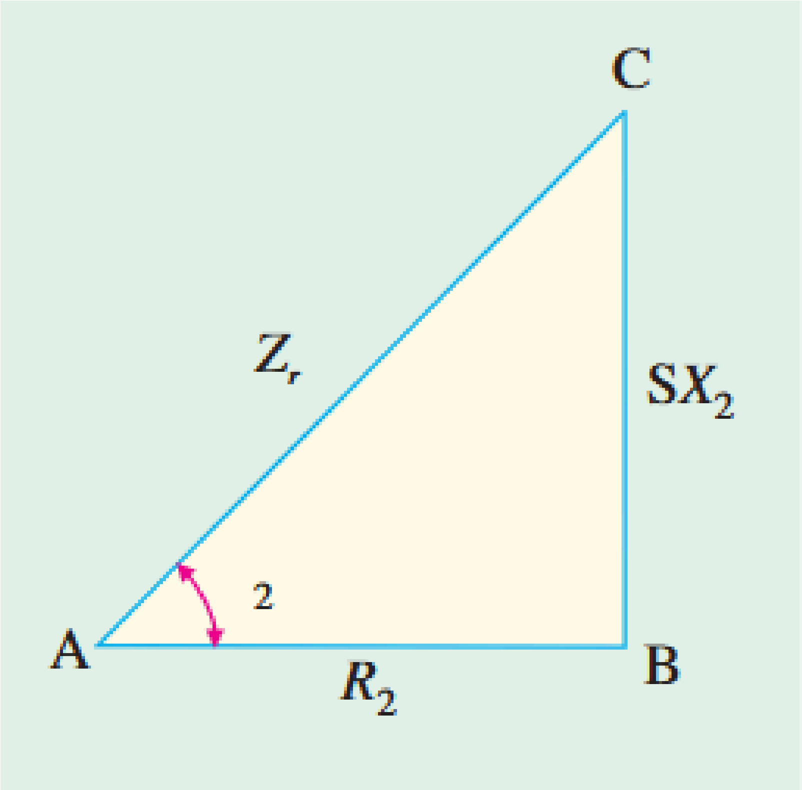


Fig. 34.20

where k_1 is another constant. Its value can be proved to be equal to $3/2\,\pi N_s$ (Art. 34.38). Hence, in that case, expression for torque becomes

$$T = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2\,R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2\,R_2}{Zr^2}$$

At standstill when $s = 1$, obviously

$$T_{st} = \frac{k_1E_2^2\,R_2}{R_2^2 + X_2^2}\left(\text{or} = \frac{3}{2\pi N_S} \cdot \frac{E_2^2R_2}{R_2^2 + X_2^2}\right) \quad \text{the same as in Art. 34.13.}$$

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Condition for Maximum Torque Under Running Conditions

The torque of a rotor under ***running*** conditions is

$$T = \frac{k\Phi s E_2 R_2}{R_2^2 + (sX_2)^2} = k_1 \frac{sE_2^2\, R_2}{R_2^2 + (sX_2)^2} \quad \text{...}(i)$$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip s and then putting it equal to zero. However, it is simpler to put $Y = \frac{1}{T}$ and then differentiate it.

$$\therefore \quad Y = \frac{R_2^2 + (sX_2)^2}{k\,\Phi\, sE_2R_2} = \frac{R_2}{k\,\Phi\, sE_2} + \frac{sX_2^2}{k\Phi\, E_2R_2}\,;\; \frac{dY}{ds} = \frac{-R_2}{k\Phi\, s^2E_2} + \frac{X_2^2}{k\Phi E_2R_2} = 0$$

$$\therefore \quad \frac{R_2}{k\,\Phi\, s^2E_2} = \frac{X_2^2}{k\,\Phi\, E_2R_2} \quad \text{or} \quad R_2^{\,2} = s^2 X_2^{\,2} \text{ or } R_2 = sX_2$$

Hence, torque under ***running condition*** is maximum at that value of the slip s which makes rotor reactance per phase equal to rotor resistance per phase. This slip is sometimes written as s_b and the maximum torque as T_b.

Condition for Maximum Torque Under Running Conditions

Slip corresponding to maximum torque is $s = R_2/X_2$

Putting $R_2 = sX_2$ in the above equation for the torque, we get

$$T_{max} = \frac{k\,\Phi\,s^2\,E_2 X_2}{2\,s^2\,X_2^2}\left(\text{or } \frac{k\,\Phi\,sE_2R_2}{2R_2^2}\right) \text{or } T_{max} = \frac{k\,\Phi\,E_2}{2\,X_2}\left(\text{or } \frac{k\,\Phi\,sE_2}{2\,R_2}\right) \qquad \text{...}(ii)$$

Substituting value of $s = R_2/X_2$ in the other equation given in (i) above, we get

$$T_{max} = k_1 \frac{(R_2/X_2)\,.\,E_2^2\,.\,R_2}{R_2^2 + (R_2/X_2)^2\,.\,X_2^2} = k_1 \frac{E_2^2}{2\,X_2}$$

Since, $k_1 = 3/2\pi N_s$, we have $T_{max} = \dfrac{3}{2\pi N_s} \cdot \dfrac{E_2^2}{2X_2}$ N-m

From the above, it is found

1. *that the maximum torque is independent of rotor resistance as such.*
2. *however, the speed or slip at which maximum torque occurs is determined by the rotor*

Condition for Maximum Torque Under Running Conditions

resistance. As seen from above, torque becomes maximum when rotor reactance equals its resistance. Hence, by varying rotor resistance (possible only with slip-ring motors) maximum torque can be made to occur at any desired slip (or motor speed).

3. *maximum torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.*
4. *maximum torque varies directly as the square of the applied voltage.*
5. *for obtaining maximum torque at starting (s =1), rotor resistance must be equal to rotor reactance.*

Relation Between Torque and Slip

$$T = \frac{k\,\Phi\, sE_2 R_2}{R_2^2 + (sX_2)^2}$$

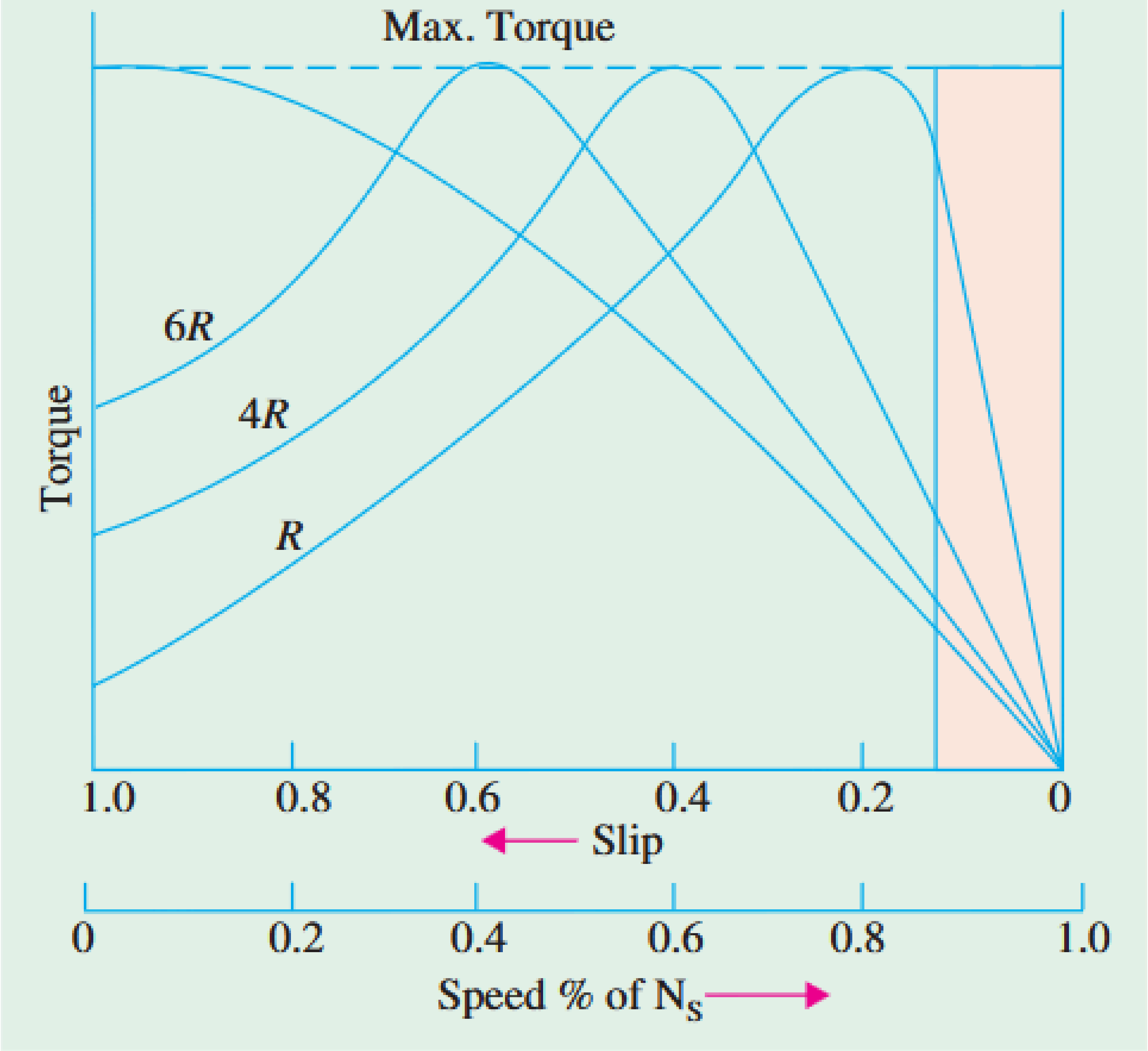
It is clear that when $s = 0$, $T = 0$, hence the curve starts from point O.

At normal speeds, close to synchronism, the term $(s\,X_2)$ is small and hence negligible *w.r.t.* R_2.

$$\therefore \qquad T \propto \frac{s}{R_2}$$

or $\qquad T \propto s$ if R_2 is constant.

Hence, for low values of slip, the torque/slip curve is approximately a straight line. As slip increases (for increasing load on the motor), the torque also increases and becomes maximum when $s = R_2/X_2$. This torque is known as '***pull-out***' or '***breakdown***' torque T_b or stalling torque. As the slip further increases (*i.e.* motor speed falls) with further increase in motor load, then R_2 becomes negligible as compared to $(sX_2.)$. Therefore, for large values of slip



$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

Full-load Torque and Maximum Torque

Let s_f be the slip corresponding to full-load torque, then

$$T_f \propto \frac{s_f R_2}{R_2^2 + (s_f X_2)^2} \quad \text{and} \quad T_{max} \propto \frac{1}{2 \times X_2}$$

$$\therefore \quad \frac{T_f}{T_{max}} = \frac{2 s_f R_2 X_2}{R_2^2 + (s_f X_2)^2}$$

Dividing both the numerator and the denominator by X_2^2, we get

$$\frac{T_f}{T_{max}} = \frac{2 s_f \cdot R_2 / X_2}{(R_2 / X_2)^2 + s_f^2} = \frac{2 a s_f}{a^2 + s_f^2}$$

where $a = R_2/X_2$ = resistance/standstill reactance*

Starting Torque and Maximum Torque

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2}$$

$$T_{\max} \propto \frac{1}{2\,X_2}$$

$$\therefore \quad \frac{T_{st}}{T_{\max}} = \frac{2R_2X_2}{R_2^2 + X_2^2} = \frac{2R_2 / X_2}{1 + (R_2 / X_2)^2} = \frac{2a}{1 + a^2}$$

where $a = \dfrac{R_2}{X_2} = \dfrac{\text{rotor resistance}}{\text{stand still reactance}}$ per phase*

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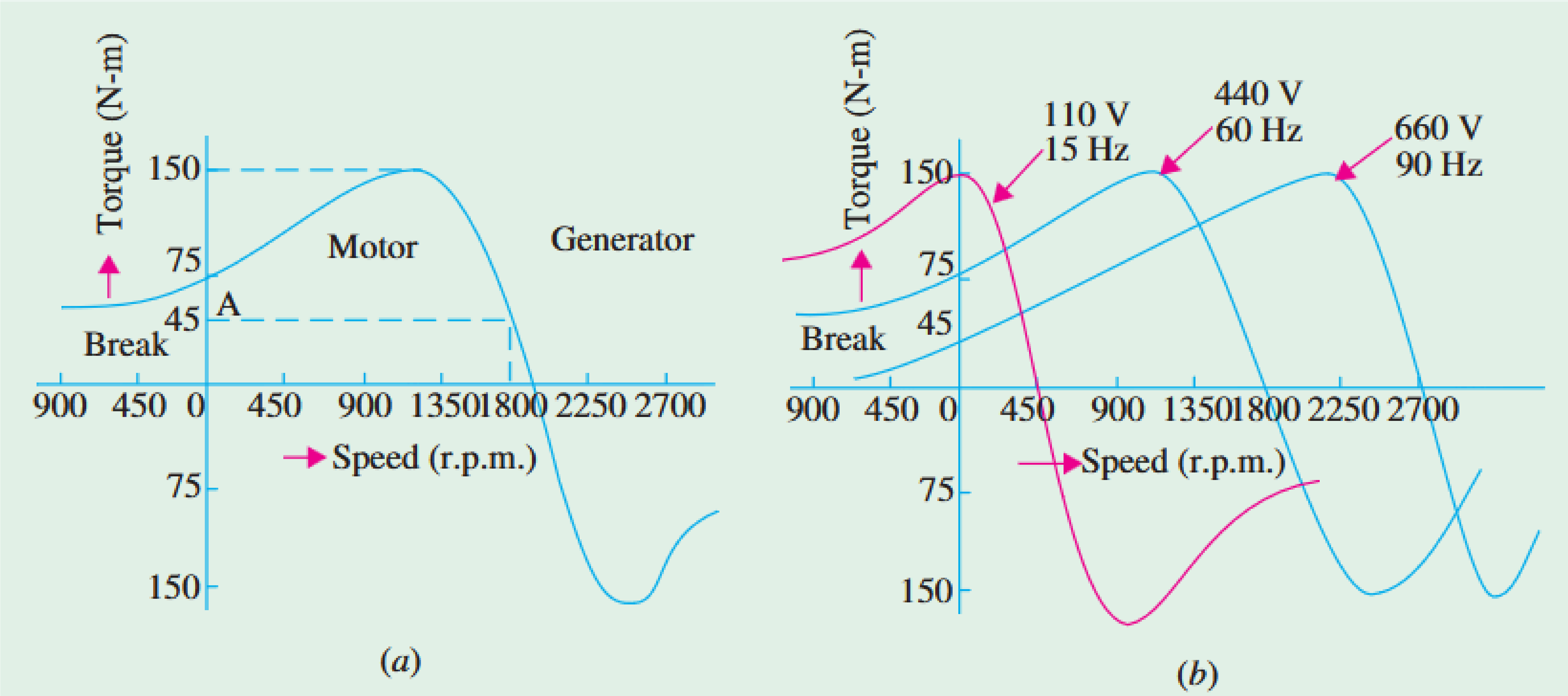
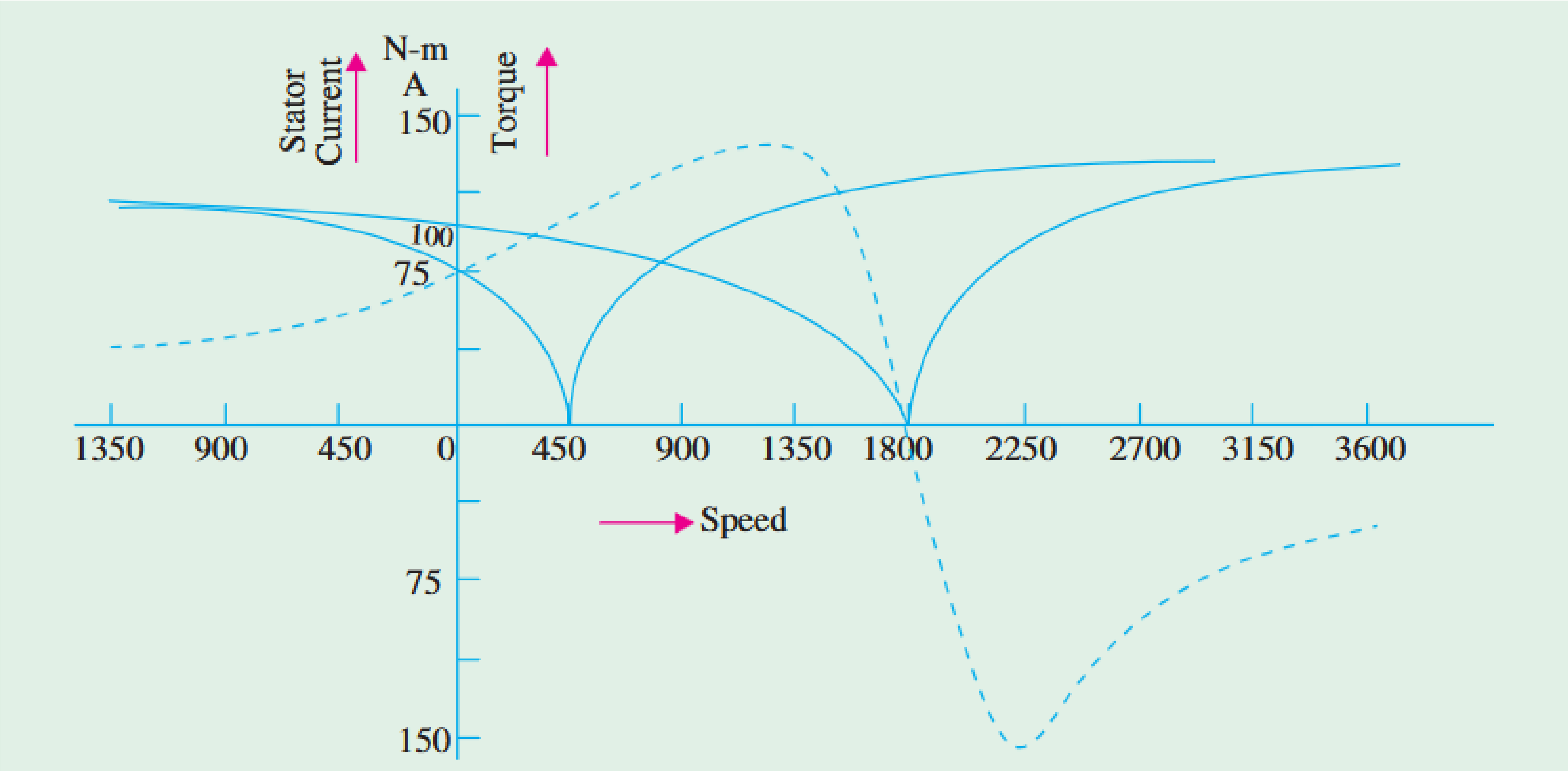
Shape of Torque/Speed Curve

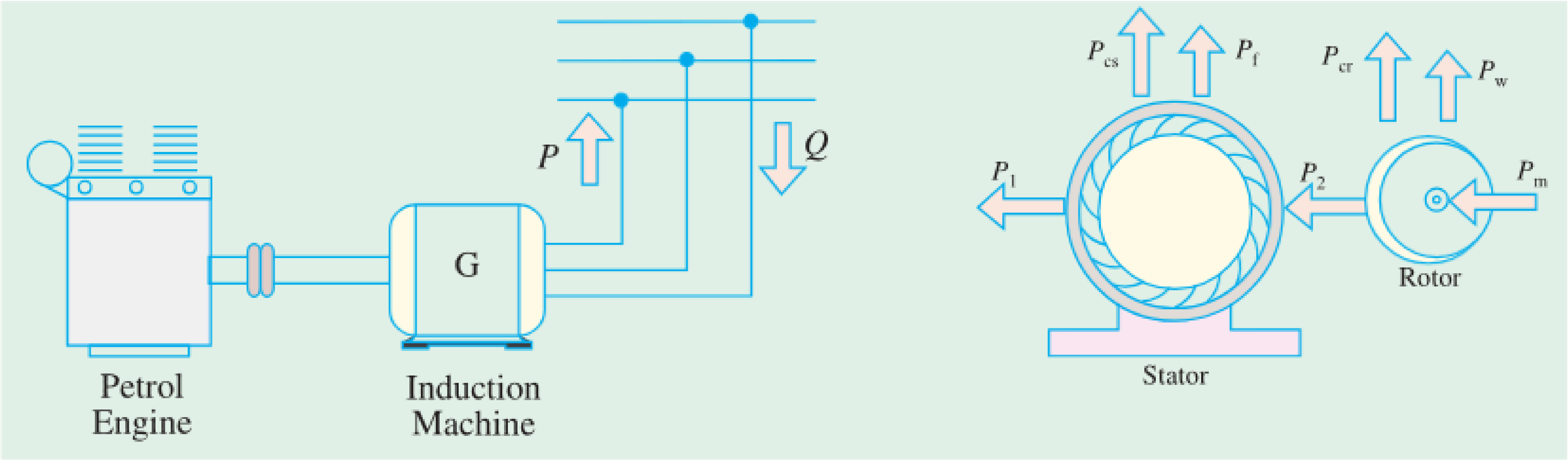


Current/Speed Curve of an Induction Motor



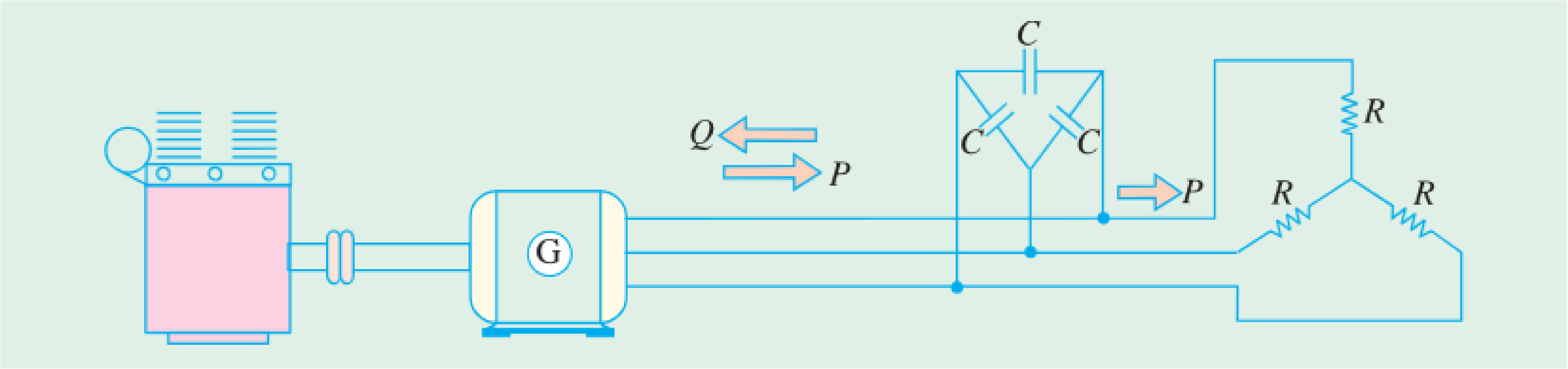
Induction Motor Operating as a Generator

When run *faster than* its synchronous speed, an induction motor runs as a generator called a ***Induction generator***. It converts the mechanical energy it receives into electrical energy and this energy is released by the stator (Fig.). Fig. shows an ordinary squirrel-cage motor which is driven by a petrol engine and is connected to a 3-phase line. As soon as motor speed exceeds its synchronous speed, it starts delivering ***active*** power P to the 3-phase line. However, for creating its own magnetic field, it absorbs *reactive* power Q from the line to which it is connected. As seen, Q flows in the ***opposite*** direction to P.



Induction Motor Operating as a Generator

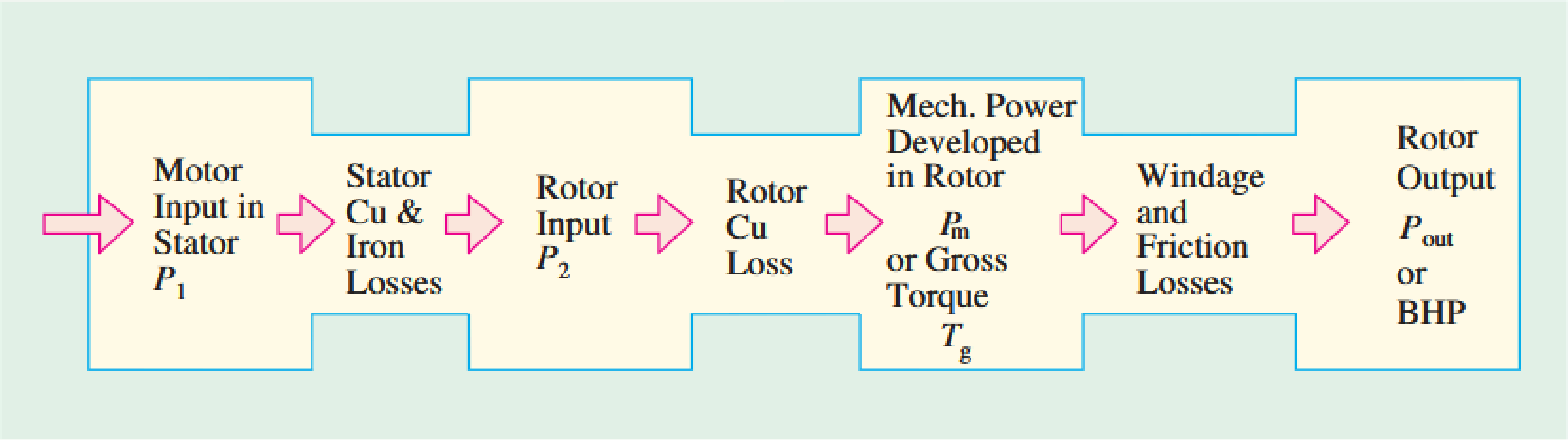
The active power ***is directly proportional to the slip*** above the synchronous speed. The reactive power required by the machine can also be supplied by a group of capacitors connected across its terminals (Fig.). This arrangement can be used to supply a 3-phase load without using an external source. The frequency generated is slightly less than that corresponding to the speed of rotation.



Power Stages in an Induction Motor

Stator iron loss (consisting of eddy and hysteresis losses) depends on the supply frequency and the flux density in the iron core. It is practically constant. The iron loss of the rotor is, however, negligible because frequency of rotor currents under normal running conditions is always small. Total rotor Cu loss = $3 \ I_2^2 R_2$.

Different stages of power development in an induction motor are as under :



Torque Developed by an Induction Motor

An induction motor develops gross torque T_g due to gross rotor output P_m (Fig). Its value can be expressed either in terms of rotor input P_2 or rotor gross output P_m as given below.

$$T_g = \frac{P_2}{\omega_s} = \frac{P_2}{2\pi N_s} \quad \text{...in terms of rotor input}$$

$$T_g = \frac{P_m}{\omega} = \frac{P_m}{2\pi N} \quad \text{... in terms of rotor output}$$

The shaft torque T_{sh} is due to output power P_{out} which is less than P_m because of rotor friction and windage losses.

$$\therefore \quad T_{sh} = P_{out}/\omega = P_{out}/2\pi N$$

The difference between T_g and T_{sh} equals the torque lost due to friction and windage loss in the motor.

In the above expressions, N and N_s are in r.p.s. However, if they are in r.p.m., the above expressions for motor torque become

$$T_g = \frac{P_2}{2\pi N_s/60} = \frac{60}{2\pi} \cdot \frac{P_2}{N_s} = 9.55 \frac{P_2}{N_s} \text{ N-m}$$

$$= \frac{P_m}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_m}{N} = 9.55 \frac{P_m}{N} \text{ N-m}$$

$$T_{sh} = \frac{P_{out}}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_{out}}{N} = 9.55 \frac{P_{out}}{N} \text{ N-m}$$

Induction Motor Torque Equation

The gross torque T_g developed by an induction motor is given by

$$T_g = P_2 / 2\pi N_s \qquad \text{—}N_s \text{ in r.p.s.}$$

$$= 60\, P_2 / 2\pi N_s = 9.55\, P_2 / N_s \qquad \text{—}N_s \text{ in r.p.m.}$$

Now, $\quad P_2 = \text{rotor Cu loss}/s = 3I_2^2 R_2 / s$

As seen from Art. 34.19, $\quad I_2 = \dfrac{sE_2}{\sqrt{R_2^2 + (s\,X_2)^2}} = \dfrac{sKE_1}{\sqrt{R_2^2 + (s\,X_2)^2}}$

where K is rotor/stator turn ratio per phase.

$$\therefore \quad P_2 = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s\,X_2)^2} \times \frac{1}{s} = \frac{3\,s\,E_2^2 R_2}{R_2^2 + (s\,X_2)^2}$$

Also, $\quad P_2 = 3 \times \dfrac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s\,X_2)^2} \times \dfrac{1}{s} = \dfrac{3\,s\,K^2 E_1^2 R_2}{R_2^2 + (s\,X_2)^2}$

$$\therefore \quad T_g = \frac{P_2}{2\pi N_s} = \frac{3}{2\pi N_s} \times \frac{s\,E_2^2 R_2}{R_2^2 + (s\,X_2)^2} \qquad \text{— in terms of } E_2$$

or $\quad = \dfrac{3}{2\pi N_S} \times \dfrac{s\,K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \qquad \text{— in terms of } E_1$

Here, E_1, E_2, R_2 and X_2 represent phase values.

In fact, $3\,K^2/2\,\pi N_s = k$ is called the constant of the given machine. Hence, the above torque equation may be simplified to

$$T_g = k \frac{s\,E_1^2 R_2}{R_2^2 + (sX_2)^2} \qquad \text{— in terms of } E_1$$

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Mathematical Problems on Induction Motor

Mathematical problems related to transformers will be practiced and solved during classroom sessions. Problems from the prescribed reference book will be addressed, and additional practice materials will be provided to enhance understanding and proficiency.